ADVANCES IN NONLINEAR MODEL REDUCTION & PARAMETRIC MODELLING USING THE PROPER GENERALIZED DECOMPOSITION

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Summary In this work, we present recent advances and promising developments in the application of the Model Order Reduction (MOR) method known as Proper Generalized Decomposition (PGD) to the simulation of nonlinear thermo-mechanical problems and the construction of parametric solutions. The PGD is a powerful a priori model reduction technique based on the separation of the problem variable that is particularly well suited for computing parametric solutions. Its direct application to nonlinear problems is however difficult reducing thereby the attractiveness of the method for non-academic applications. We investigate the potential of the asymptotic numerical method coupled to the PGD for the efficient solution of non-linear parametric models. A particular emphasis is put on real-time and optimization applications.

INTRODUCTION

Despite constant impressive advances in numerical analysis, discretization techniques, computer science and high performance computing, many thermo-mechanical problems remain today intractable using classical simulation approaches. This can stem either from the sheer complexity of the problem or from time constraints on the solution method. As advanced numerical simulations strive to be on the limit of what modern computers allow, they often can generate results at a maximum pace that is totally incompatible with the requirements of e.g. virtual reality or real time applications. Additionally the increase in available computing power is often balanced by a corresponding increase in the complexity and/or size of the model to be solved. Among the many cases where these limitations occur we distinguish the following cases:

1. Optimization and inverse problems require the computation of a large number of solutions for many different values of the identified/optimized parameters. The number of solutions to compute increases further when derivatives of the function to optimize with respect to the parameters have to be evaluated. In the case of large problems this large number of direct problems constitute the dominant computing cost. Such costs make the use of advanced optimization algorithms (genetic algorithms, simulated annealing…), or simply the computation of high dimensional gradients prohibitive. An alternative of course lies in the use of approximate surrogate models. Such models however need to be identified which is a non-trivial task as the "true model" is often large and nonlinear.
2. For real-time and virtual reality applications based on a complex model, the simulation has to be fast enough to incorporate in real time external inputs. Surrogate models are therefore a common alternative, with the drawbacks discussed before. Should one wish to pre-compute the true model for "all" possible input scenarios, modern mesh-based techniques would face at least two major obstacles:
   - It would take ages to a priori simulate all scenarios.
   - The mere storage of the outputs of these scenarios would be unfeasible.

These obstacles call for a change of paradigm for the numerical simulations that are to be integrated into virtual/augmented reality environments, real time applications and fast optimization. The recently proposed numerical method called Proper Generalized Decomposition (PGD) provides such change of paradigm.

One of the main features of the PGD is its ability to solve highly multidimensional models while alleviating the associated pitfalls often referred to as "the curse of dimensionality" and maintaining very low computational and memory costs. In the next section, we briefly review the features of the PGD and show its relevance with respect to the above-mentioned obstacles.

PROPER GENERALIZED DECOMPOSITION

Ammar et al. introduced the PGD [1] in the framework of computational rheology for solving models originating from the kinetic theory description of viscoelastic fluids. At the core of the PGD, lies the separated representation of the unknown field as a finite sum of functional products involving functions defined on lower dimensionality spaces. For example a scalar field u(x,y,z,t, α , β) defined on a space of dimension 6 can be approximated in the following way:

$$u(x,y,z,t,\alpha_1,\alpha_2) = \sum_{i=1}^{N} X_i(x,y) \cdot Z_i(z) \cdot T_i(t) \cdot A_i(\alpha) \cdot B_i(\beta)$$

The specific partition of the problem variables in the above functional product depends on the problem being solved. A similar partition of space and time variables has been proposed in the early 80s by P. Ladeveze in the framework of the LATIN Method [2].

The PGD incrementally computes the different terms of the finite sum, making use of an appropriate linearization strategy to compute the different factors of the functional product. Using this approach, high dimensionality problems

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are never solved and as soon as the solution exhibits some degree of regularity, the number N of terms in the finite sum is limited (from a few tens to some hundreds) hence the low computational and memory costs. The two following features of the method particularly interest us:

- The separation of variables depicted above can be applied to the space coordinates in order to break down the complexity of the problem. Many relevant engineering problems are formulated on degenerate domains for which one can separate at least one space coordinate from the others. This is the case for example in plate or shell domains where one can separate the out-of-plane coordinate from the in-plane coordinates. This features allows the solution of the full 3D problem (e.g. thermal, mechanical) at a numerical cost scaling like 2D without resorting to a priori assumption on the unknown field like in classical plate theories [3].
- The method allows the introduction of many extra-coordinates to the problem. Such coordinates are not restricted to the classical space-time variables but can be material, process or design parameters of the simulated system.

These two features are clearly of interest for problems needing a fast solution. The key point for real time and optimization applications is that the PGD solution needs to be computed once in an offline step, possibly on a powerful computing platform. During the online step (i.e. the real time application), one only needs to particularize the different variables depending on the scenario and evaluate the solution or its derivative, which is computationally very cheap [4].

For solving non-linear problems with the PGD, Ladeveze has proposed in the framework of the LATIN method specific solvers for non-parametric thermo-mechanical models [2]. More recently, Ammar et al. proposed alternative strategies that can be used for parametric models [5]. The asymptotic numerical method [6] proposed by Potier-Fery is an appealing strategy for solving non-linear problems with the PGD since it essentially requires many solutions of a unique linear problem with a varying right-hand-side. Moreover its generality is an asset when incorporating many extra-coordinates with different physical meanings.

Preliminary results on the combination of the asymptotic numerical method with the PGD are promising. As an example, let us consider the following heat transfer problem in a square domain with sides of length 2:

$$-k\,\Delta\, u - 1 + \lambda u^2 = 0$$

Here u is the temperature field, k the heat diffusivity and λ is the non-linear loading parameter used in the asymptotic expansion. We impose the Dirichlet boundary conditions $u(\Gamma) = 2$ on all the boundary Γ of the domain. A parametric solution $u(x, y, k)$ is computed with the PGD. Figure 1, shows the convergence of u at the centre of the domain as a function of λ and of the order of the asymptotic expansion.

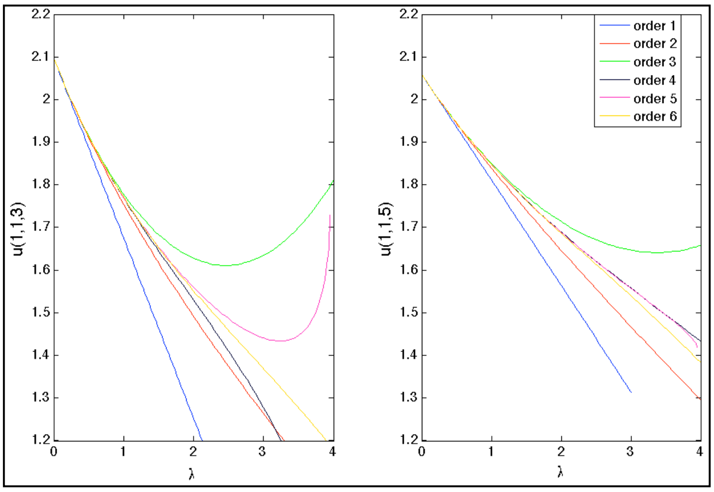

Figure 1: Convergence of the PGD parametric solution as a function of the loading parameter at the centre of the domain for different values of the thermal diffusivity and different orders of expansion of the nonlinearity

CONCLUSIONS

The Proper Generalized Decomposition is a powerful model order reduction technique that yields significant computing time saving when applied to degenerate domains such as plate or "extruded domains" which are common in engineering problems. Furthermore its ability to solve parametric models allows the efficient a priori solution of a model for many different simulation scenarios (loading conditions, material & geometrical parameters…). These features make it very appealing for real-time applications. The use of the asymptotic numerical method coupled to the PGD offers a promising way for the solution of parametric models.

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