A SIMPLE NONLINEAR CUTTING MODEL FOR THE QUICK QUALITATIVE DESCRIPTION OF CHIP FORMATION

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Summary A simple thermo-mechanical model of the cutting process is introduced in this paper. The results – that were verified experimentally – can be used for the quick qualitative prediction of the chip type.

INTRODUCTION

The accurate description of processes is essential for the development of manufacturing technologies. Although several finite element software packages are available for the modeling of cutting processes, the overview of the effects of parameter changes is not easy with them and the compilation of the FEM model usually requires significant efforts. Another successful approach is the use of simple empirical formulas for the calculation of certain characteristics of the cutting process. For example, the use of the so-called three quarter rule is usual in publications that concentrate on the description of the regenerative effect [1]. Our goal is to find a trade-off between these two approaches and set up a simple cutting model that is able to the qualitative description of the process. Our basic nonlinear thermo-mechanical model comprises 4 ordinary differential equations. This model can be extended simply to the description of the effects of varying cutting speed and varying depth of cut. We believe that the simplicity of the – experimentally validated – model and the relatively low number of parameters can help in the quick overview of the cutting process.

THERMO-MECHANICAL MODEL

Our model is based on the widely used assumption that the deformation is concentrated into one or more deformation bands (shear zones) during cutting. As Fig. 1 shows, we splitted the shear zone into three layers of equal thickness δ. There is no plastic deformation in layer 0 which plays a role in the thermal processes only. The plastic deformation occurs in the other two layers. We assumed that the rake angle α of the tool equals the angle of the shear plane Φ, and we ignored the strain-hardening[b)]. We also supposed that thermal conduction occurs only on the boundary surfaces of the layers. Exploiting these assumptions, two differential equations can be obtained that describe the mechanical balance of plastic shear stresses $\tau_{1,2}$ in the deformation layers 1 and 2 with the normal stress δ acting on the face of the tool. Three energy equations describe the heat flow among the three layers of temperatures T_0, T_1, and T_2 [3]. Since we were able to express τ_1 by τ_2, our model comprises the following dimensionless ordinary differential equations:

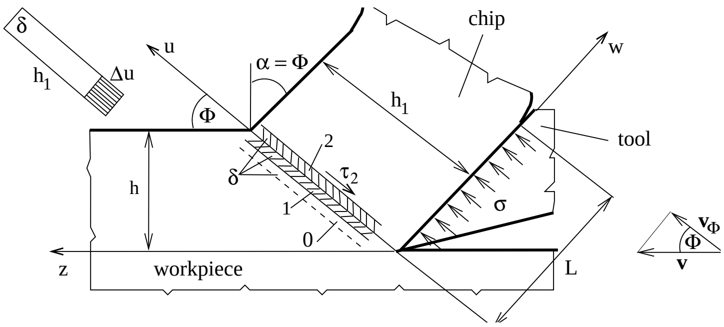

Figure 1. The thermo-mechanical model

$$\dot{\tau}_2 = f\left(1-(F_1+F_2)\right) \tag{1}$$

$$\dot{T}_0 = \zeta(T_1-2T_0)-f\xi T_0 \tag{2}$$

$$\dot{T}_1 = f\Big(\eta(\tau_2 p+1-p)F_1(\tau_1,T_1)-\xi(T_1-T_0)\Big)-\zeta(2T_1-T_2-T_0) \tag{3}$$

$$\dot{T}_2 = \eta f\tau_2 F_2(\tau_2,T_2)-(f\xi+\zeta)(T_2-T_1) \tag{4}$$

F_1 and F_2 denote the deformation velocities in layers 1 and 2, that depend on only three material parameters. p characterizes the geometry of chip formation, K denotes the time scale, ζ is the dimensionless velocity of the chip. The amount of heat produced during plastic deformation is described by η, while ξ is related to both thermal conduction and technological conditions. f denotes the ratio of the actual cutting speed to a reference speed. The expressions of these nondimensional parameters can be found in [3, 5]. The equilibrium solutions of this system correspond to the formation of continuous chips, periodic solutions indicate segmented chips, while the occurrence of chaotic solutions means that the chip becomes aperiodic.

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b) These simplifying assumptions were disregarded in our new, refined model [2].

BIFURCATION CHARTS

We explored the structure of the solutions by bifurcation analysis. For this purpose, we numerically searched for stable equilibrium solutions. The change of the position of equilibrium states was followed by the continuation and bifurcation software AUTO. The bifurcation points were also detected during this process. We found that the parameters of period doubling bifurcations follow each other according to the Feigenbaum ratio. Thus, we were able to estimate the borders of the parameter domain of chaotic solutions [4]. The results of this calculation are shown in Figure 2.

It is easy to extend our model to the description of various additional effects [2, 5]. For example, the oscillation of the tool or the workpiece can be taken into account by exchanging the coefficient f in Equations (4) to the expression of the effective cutting velocity. If the nondimensional velocity of the tip of the tool in the direction of cutting is $\dot{z}$ and the reference velocity is denoted by V_0, the expression of the effective velocity becomes $f\left(1 - \frac{\dot{z}}{V_0 f}\right)$. By supplementing the set of equations with the simplest possible differential equation of the oscillating tool $\ddot{z} = \tau_2 - Az$, we obtain a 6D model. Certainly, the oscillation of the tool can be modeled in a more refined way, too, e.g., by taking into account the regenerative effect.

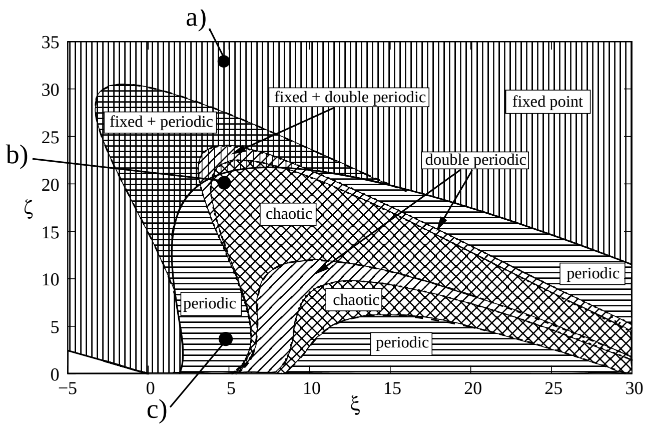

Figure 2. Two-parameter bifurcation diagram.

We found that due to the interaction between the oscillating workpiece and the formation of chips, the solutions became more complex than in the case of the 4D model. For example, the original equilibrium solution turned to small amplitude chaotic solution if the tool was assumed to be elastic. This result is in agreement with the practical experience, i.e., that the chip thickness varies slightly and irregularly in the case of continuous chips, too.

If the amplitude of the vibrations exceeds a critical value, the tool may temporarily leave the workpiece. The forces that arise when the tool starts to cut again are very large and may lead to the damage of the tool or the workpiece. We numerically determined the domains on the plane of parameters f (cutting speed) and A (tool stiffness) where this situation may occur. The red points of Figure 3 denote these dangerous parameter domains, while the tool did not leave the workpiece in the domains denoted by green.

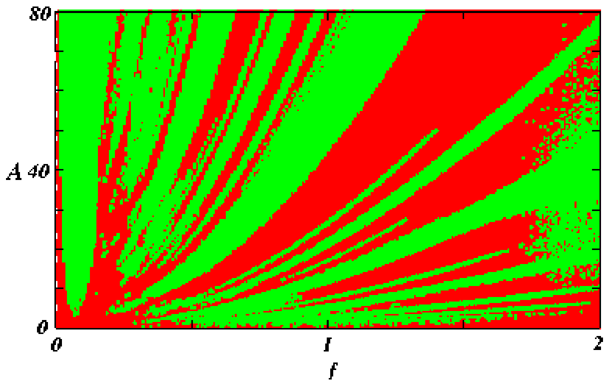

Figure 3. Safe and dangerous parameter domains.

CONCLUSIONS

A new model for the description of large plastic deformation of metals was introduced for the case of cutting. A bifurcation analysis was performed and chaotic solutions were found. The model was applied to the case of an elastic, oscillating tool and the domains where the tool may leave the workpiece were determined. Although the regenerative effects are not considered yet, the model can be coupled with regenerative models. We believe that this simple model makes a quick overview of the sources of vibrations during the cutting process possible. Moreover, it can provide help during the design and evaluation of preliminary experiments and measurements that are necessary for the preparation of the production process. In the case of large simulation software, our model may support the design or can be used for the determination of certain parameters in applications.

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