DESCRIPTION OF PERMEABILITY ANISOTROPY OF GRANULAR MATERIALS WITH ANISOTROPIC POROSITY DISTRIBUTION

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Summary Fluid flow in granular materials is usually direction dependent owing to tortuosity associated with the internal structures of the materials. This paper presents an approach describing the permeability anisotropy of saturated granular soils using a structural measure describing the anisotropic distribution of pore voids. The permeability tensor is derived based on volume averaging approach through which the tortuosity tensor is related to the directional porosity distribution. The analytical results are found generally agree with experimental data of Ottawa sand and glass beads.

INTRODUCTION

Since the pioneering work of Kozeny [1], numerous studies have been performed to correlate the permeability of saturated porous media with its physical properties, including porosity, pore size and structures, mean particle size and particle size distributions. In the various models in the literature, tortuosity is an important quantity that is used to bridge the pore structure and the macroscopic permeability [2]. More recently, advance digital image analysis techniques including X-ray microtomography are used for 3D imaging of solid (or void) structures, from which the tortuosity can be estimated based on geodesic reconstruction. Based on the reconstructed void structures, flow in the porous media can be directly simulated using the Lattice-Boltzmann method (LBM) and the permeability can be determined correspondingly [3-5]. In this paper, a different approach is investigated to describe the permeability anisotropy of saturated granular soils using a volume averaging approach, by incorporating a continuum measure of internal structural disorder identified with the directional distribution of porosity/void ratio of the material. Thus, even though the formulation remains phenomenological in nature, it provides some insight into the void structure on a micro-scale.

FLOW IN REV OF GRANULAR MATERIALS

The flow of water in a saturated (two-phase) granular material is described using an approach of local volume averaging taking into account mass balance on both the macro- and micro-levels. For a unit sphere enclosing a representative volume of the material as shown in Figure 1, let S_0 be the circumference area of the sphere encompassing the REV, S_{ss} the area around the REV that intersects the particles, S_{ff} the area around the REV that cross the fluid, and S_{fs} the total area of the interfaces between particles and the fluid. The total area enclosing the fluid is then $S_{0f} = S_{ff} + S_{fs}$. Following [2], the macroscopic balance equations of the fluid in a porous medium is expressed as

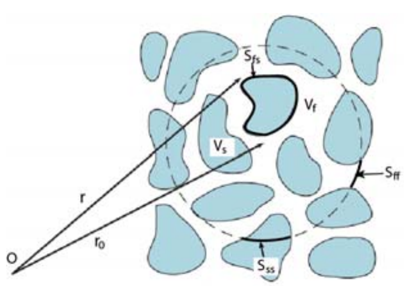

Figure 1: A REV in granular soils

$$\bar{\mu}_f^f \alpha_{ij} C_f n \frac{\bar{V}_{fj}^f - \bar{V}_{sj}^s}{R_f^2} = -nT_{fij}^* \left(\frac{\partial \bar{p}^f}{\partial x_j} + \bar{\rho}^f g \frac{\partial z}{\partial x_j} \right) \tag{1}$$

where $\overline{(\bullet)}^{\alpha}$ stands for the volumetric average of phase α, μ_f is the dynamic viscosity of the fluid, C_f a shape factor related to pore size, while V_i, ρ and p are the velocity, density and pressure respectively. Moreover, the repeated indexes denote summation. The macroscopic quantities in Eq.(1) are the porosity ϕ of the REV, the hydraulic radius R_f defined as the ratio volume of the fluid and the interface area of the fluid and the solid phase in the REV, the tortuosity tensor defined as $T^*_{fij} = (1/V_f)\int_{S_{ff}} \dot{x}_i n_{fi} dS$, with $\mathbf{n}_f$ being the unit normal vector of S_{ff}. α_{ij} is a second-rank tensor depending on the configuration of solid particles and is defined in [2] as $\alpha_{ij} = \delta_{ij} - (1/S_{fs})\int_{S_{fs}} n_{fi} n_{fj} dS$.

TORTUOSITY TENSOR AND DIRECTIOANL DISTRIBUTION OF VOIDS

The anisotropic nature of the material is characterized using the spatial distribution of pore voids described by a probability density function [6, 7], which results in a representation of the distribution of the volume fraction of voids, or porosity $\phi(\mathbf{n})$,

$$\phi(\mathbf{n}) = \phi_0(1 + \Omega_{ij} n_i n_j + \Omega_{ijkl} n_i n_j n_k n_l + \cdots) \tag{2}$$

or more simply $\phi(\mathbf{n}) = \phi_0(1 + \Omega_{ij} n_i n_j)$, in which ϕ_0 is the average porosity of the material, Ω_{ij} and Ω_{ijkl} are symmetric traceless tensors. As such, the tortuosity tensor is determined as

$$T^*_{fij} = \frac{\phi_A}{4\pi}\int_S (\delta_{kl} + \Omega_{kl}) n_k n_l n_i n_j dS \tag{3}$$

where $\phi_A = S_{ff}/S_0 = (1/4\pi)\int_S \phi(\mathbf{n}_A) dS$ is the areal porosity on S_0 and depends on the directional distribution of porosity in the tangential direction of the REV.

ANISOTROPIC PERMEABILITY AND DIRECTIOANL DISTRIBUTION OF VOIDS

The influence of solid particle configuration on the anisotropy of permeability is reflected in the second-rank tensor α_{ij} defined in [2] as $\alpha_{ij} = \delta_{ij} - (1/S_{fs})\int_{S_{fs}} n_{fi} n_{fj} dS$. This integral in the expression of α_{ij} expresses the total static moment of the oriented *sf*-interfaces with respect to the planes passing the centroid of the REV. In this work, the integral on S_{fs} in the expression of α_{ij} is determined by the hydraulic radius Δ_f of the REV as

$$\alpha_{ij} = \frac{1}{3}(2\delta_{ij} + T^*_{fij}) \tag{4}$$

which also depends on the spatial distribution of pore voids in the material. For isotropic materials with $\phi(\mathbf{n}) = \phi_0 \delta_{ij} n_i n_j$, it can be shown that $\phi_A = 1/3$, $T^*_{fij} = \phi_A \delta_{ij}$ and $\alpha_{ij} = (7/9)\delta_{ij}$, which are very similar to the results in [2].

EXPERIMENTAL INVESTIGATION

A series of laboratory tests using standard Ottawa sand and glass beds are carried out. Several methods are used to fabricate homogeneous specimens with varying internal structure (or fabric), and the permeability along different directions relative to the principal axes are measured. Thereafter, efforts are made using digital image analysis to obtain the void distribution in the specimens with different sample preparation methods. It is found that taking into account the directional variation of tortuosity through the directional porosity yields reasonable results for anisotropic permeability when comparing the theoretical analyses and experimental results.

CONCLUSIONS

This paper presents an approach describing the permeability anisotropy of saturated granular soils using a structural measure describing the anisotropic distribution of pore voids. Both the tortuosity and the configuration of solid grains are made functions of directional variation of pore voids. When assuming all voids are connected, which is true for granular soils, the permeability tensor is derived based on a volume averaging approach. The analytical results are found generally agree with experimental data of Ottawa sand and glass beads.

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