SPREADING OF A COHESIVE GRANULAR MATERIAL ON A SMOOTH INCLINED PLANE SUBMITTED TO VIBRATION

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Summary What are the influences of vibrations on rheological properties for dry and cohesive bulk solid? After studying an angular particle sliding down a transversally vibrated smooth plane, we continue our research by studying a thin layer of dry and cohesive sliding on a transversally vibrated smooth plane. We introduce the vibration force on Saint Venant equations to simulate the spreading of granular mass that we compare to experimental results.

INTRODUCTION

Granular matter is usually used by many industrial activities: civil building industry (cement, sand, gravel), the pharmaceutical industry (excipient powders), agro-chemical (seeds, nutriments), food industry (wheat, starch, cereals), moulding processes (metallic powders, plastics, ceramics) in particular. The control of their implementation needs the comprehension of their flow behaviour [1]. After studying an angular particle sliding down a transversally vibrated smooth plane [2], we continue our research by studying a thin layer of dry and cohesive bulk solid sliding on a transversally vibrated smooth plane. To simulate the spreading of granular mass, Saint Venant equations are appropriated as [3, 4]. We introduce the vibration force in the simulation that we compare to experimental results.

THEORETICAL MODEL

By adding vibration component from Saint-Venant equations, the equation of a thin layer profile sliding on a transversally vibrated smooth plane (figure 1) is written as:

$$\frac{1}{g\cos(\theta)}\dot{\vec{U}} = \tan(\theta)\vec{x} - \frac{\vec{\tau}}{\rho g h\cos(\theta)} - \frac{A\omega^2}{g\cos(\theta)}\cos(\omega T+\varphi)\vec{y} - \frac{K}{\cos(\theta)}\frac{\partial h}{\partial x}\vec{x} \qquad (1)$$

With T, time, g, the gravity constant, {U, h} defining the mesh, θ, the slope of inclined plan, {A, ω, φ}, the vibration condition, {ρ, τ, K} defining the properties of the material: τ is the friction force with the plane and K, the ratio between vertical normal stress and the horizontal normal stress.
By a variable change (2), equation (1) becomes equation (3):

$$\left\{T = \sqrt{\frac{A}{g\cos(\theta)}}\,\tilde{t}\,;\, Fp = \omega\sqrt{\frac{A}{g\cos(\theta)}}\right\} \qquad (2) \qquad\qquad \dot{\tilde{\vec{U}}} = \tan(\theta)\vec{x} - \tilde{\vec{\tau}} - Fp^2\cos(Fp\,\tilde{t}+\varphi)\vec{y} - K\frac{\partial\tilde{h}}{\partial\tilde{x}}\vec{x} \qquad (3)$$

By applying the mass conservation equation and the equation (3) to a thin layer profile split into section (figure 2), we simulate its global evolution preserving the coherence of the profile.

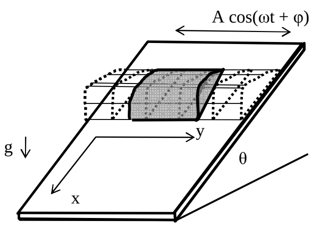

Figure 1: Draft of a dry and cohesive bulk solid sliding on a transversally vibrated smooth plane

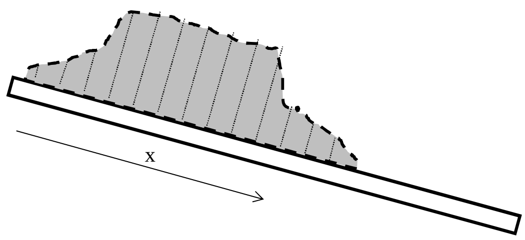

Figure 2: Thin layer profile and its mesh

This simple model depends only on known experimental conditions (slope and vibration) and parameters defining a bulk solid: μ the friction at the base and K defining a granular cohesion [4, 5]. μ and K are the adjustable parameters of our model.
From the slip condition equation, the thin layer stays jamming as long as the sum of the gravity, granular pressure and vibration forces stay lower than the friction force. On figure 3, we have plotted the phase diagram between the stick and slip mode for a thin layer.
Equations are implemented in a Matlab code using an implicit scheme. The numerical method has been tested with various classical configurations without the transverse vibration. A time step of 10^{-2} was chosen.

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$$\left(\frac{U_x}{dt}+\tan(\theta)-K\frac{\partial h}{\partial x}\right)^2+\left(\frac{U_y}{dt}-F_p^2\cos(F_p t+\varphi)\right)^2\leq\mu^2 \quad (3)$$

Slip condition at mesh level

$$\alpha=\frac{F_p^2}{\mu}=\sqrt{1-\frac{1}{\mu}\left(\tan(\theta)-K\tan(\Delta\theta)\right)^2}\quad;\beta=\frac{\tan(\theta)}{\mu} \quad (4)$$

Border between stick and slip mode

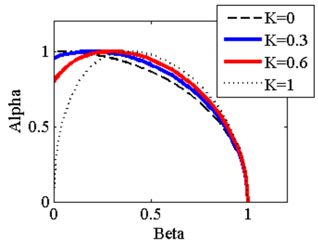

Figure 3: Phase diagram of stick and slip mode

The simulation of a thin layer profile from a real profile gives the position and velocity evolution of the meshes as well as the spreading and height of the thin layer (figure 5).

PRELIMINARY RESULTS

The experimental device uses a shaker and a mirror-polished stainless steel substrate. The granular bed is made of an alumina powder obtained by granulation, polydisperse, with a grain a size lower than 500µm, dry and cohesive (figure 4). The powder bed is deposited on the inclined plane by pluviation in a mould and flows without vibration for an angle higher than 28°. The vibrations transmitted to the plane are horizontal and sinusoidal form. For these preliminary experiments, we fixed the frequency at 30Hz, the slope at 15° and vary the relative acceleration between 0 and 1. Two fast cameras recording at 250 fps and positioning on the top and side of the flow follow the profile evolution and the spreading (or length).

Figure 5 presents an example of the experimental height and length evolution and those obtained from the model adjusting by the µ and K values to superpose the observed profile after 0.5 and 1s (figure 6). The simulated height and length values correspond with the experimental values. However, it should be underline that the parameter values (µ=tan (42,5°), K=0,2) are not in agreement with those generally cited in the literature [5] which mentions a K value at 0.3 or more for cohesive granular and an friction angle close to the spontaneous flow angle (at 28° in our case). These differences can have several origins; in particular the friction coefficient increases with the inertial number [6] and the model uses a simple friction law for bulk solid friction on inclined plane whereas the vibrations allow moving the particles between them.

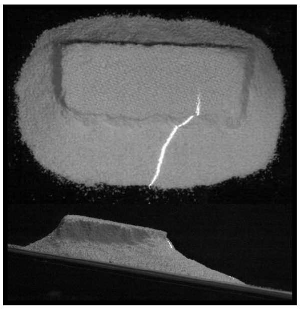

Figure 4: Powder bed morphology on top and side view

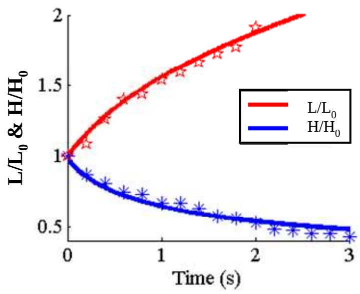

Figure 5: Experimental (dotted line) and simulated (continuous line) height and length profile {15°, 30Hz, 0.3mm} during 3s.

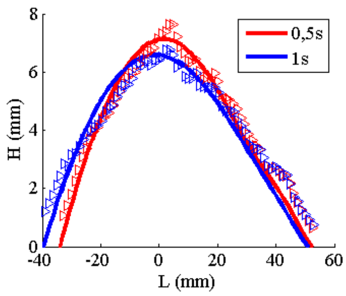

Figure 6: Experimental and simulated profile {15°, 30Hz, Γ=1}, centered on H_{max} at 0.5s and 1s.

The model allows varying experimental conditions for granular bed spreading. Measurements of the K parameter using a shear cell and other flow experiments are in progress to validate the model.

CONCLUSION

The presented 2D model gives first moments of granular material sliding on a transversally vibrated smooth plane for vibration conditions not leading to an important spreading of powder. The development, in progress, of 3D model will highlight this spreading which appears under certain vibration condition. In the long term, this model must result to define the values of parameters governing the various flow phases (stick and slip) with vibrations as well as spreading and flow velocities according to the bulk solid parameters (K and µ).

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