PATTERN FORMATION OF GRANULAR AVALANCHES SIMULATED BY PARTICLE MEHTOD WITH HYDRODYNAMICS INTERACTION

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Summary To elucidate the instability mechanism of granular avalanche, we propose a particle method considering the interaction between the granular and the fluid. Using this model, we simulate the head formation at the moving front of avalanche. It was found that the air drag destabilizes the moving front of avalanche to deform into a wavy pattern and to generate the vortex convection inside the avalanche.

INTRODUCTION

Avalanches, generally taken as a class of massive landslide phenomena, cover gravity currents and density currents, for instance, snow avalanches, debris flows, and pyroclastic flows. These flows migrate downward as a mixture of solid and fluid and form common structures, one of which is termed as the head-tail structure. Concretely, at the moving front of avalanche, a large head is formed by gathering the large materials, whereas at the rear end, an elongated tail is formed by leaving behind the smaller materials. As a factor for the formation of head-tail structure, the ratio of the air drag to the gravity is considered most relevant. For example, a granular flow consisting of the light particles like the polystyrene forms the head-tail structure as well as the wavy pattern with many heads at the moving front of avalanche [1]. In contrast, experiments using heavy particles (air drag$\ll$gravity) like the glass beads do not generate the head-tail structure although they form wavy pattern similar to ones using light particles [2]. Additionally, the experiment using light particles shows that the head size increases with the increasing particle radius. To explain these facts, several models have recently been proposed; the fluid flow model assumes an avalanching body as a mass of fluid, the mass center model assumes an avalanche as a huge particle, and so on [3, 4, 5, 6]. However, the materials constituting avalanches are granular materials such as polystyrene or glass bead and are definitely not fluid. Moreover, the interaction between particles may play a non-trivial role which is out of the scope of fluid model and one particle model. The following our models are proposed to overcome the above shortage of previous models.

MODEL

This model is roughly based on three basic assumptions; Firstly, the granular consisting of spherical (three-dimensional) particles only moves along two-dimensional surface. Secondly, only the translational motion of particles is considered, whereas the rotational motion of particles is ignored. Thirdly, as the force acting on the particle, we consider three types; (i) gravity as the dominant driving force of avalanche, (ii) repelling force between particles which causes the excluding volume effect, and (iii) drag force by fluid.
(i) Gravity; considering the density difference between the granular and the fluid, the gravity $\mathbf{F}_g$ is expressed as

$$\mathbf{F}_g = -e_y V g(\rho_p - \rho_f),$$

where e_y is the unit vector parallel to the y-axis, V is the volume of a particle, g is the gravitational acceleration, and ρ_p and ρ_f are the density of a particle and the fluid, respectively.
(ii) Repelling force; because of ignoring the rotational motion of particles, the repelling force $\mathbf{F}_c$ works on only the normal direction and is represented as the elastic spring force

$$\mathbf{F}_c = -e_\theta k_n \delta, \;\; e_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix},$$

where θ is an angle between x-axis and line passing through the center of two particles, e_θ is the rotation matrix, k_n is the spring constant, and δ is the overlap distance between two particles.
(iii) Drag force by fluid; the long-range interaction between particles though fluid is experienced by individual particles as this air drag force. Namely, by assuming that the granular motion is immediately propagated though the fluid, we use Rotne-Prager tensor [7] (eq. (1)) as the effective long-range interaction between particles.

$$J(\mathbf{r}) = \frac{1}{r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2} + \frac{2}{3}\left(\frac{a}{r}\right)^2\left(\mathbf{I} - 3\frac{\mathbf{rr}}{r^2}\right)\right], \;\; (\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j), \qquad (1)$$

where $\mathbf{I}$ is the unit tensor, $\mathbf{r}_i$ is the coordinate of ith particle, r is the interparticle distance, and a is the particle radius. Note that $\mathbf{rr}$ in eq. (1) is the matrix, in which each element corresponds to the tensor J, for instance, a element coincide with J_{xy} is expressed as $\mathbf{r}_x\mathbf{r}_y = (x_i - x_j)(y_i - y_j)$.

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Moreover, by ignoring the particle inertia, the ith particle velocity $\mathbf{v}_i$ is determined by summing the velocities derived from the force acting on itself and generated by other particles as

$$\mathbf{v}_i = \frac{\mathbf{F}_i}{6\pi\mu a} + \sum_{j\neq i}^{N} \mathbf{u}_i(j), \;\; \mathbf{u}_i(j) = \frac{1}{8\pi\mu r}\left[\mathbf{I} + \frac{\mathbf{rr}}{r^2}\right]\cdot\mathbf{F}_j + \frac{a^2}{12\pi\mu r^3}\left[\mathbf{I} - 3\frac{\mathbf{rr}}{r^2}\right]\cdot\mathbf{F}_j,$$

where $\mathbf{F} = \mathbf{F}_g + \mathbf{F}_c$, μ is the viscosity coefficient of fluid, and $\mathbf{u}_i(j)$ is the induced velocity generated by jth particle at the coordinate of ith particle.

RESULTS

Numerical simulations are conducted on a slope with a constant inclination angle ($\theta = 45°$), where particles are confined in two different kind of two-dimensional spaces depending on model; model-I) particles move along two-dimensional surface of slope, model-II) particles move within two-dimensional vertical cross-section extending over the slope. Note that, in model-I, the effect of the inclination angle is reflected in the term of gravity as $g\sin\theta$, whereas model-II assumes that the collision with the slope is described as the elastic spring force similar to the collision between particles. Both the simulations of model-I and II are carried out with $N = 2000$ particles using three different sizes (small, medium, large) of particles.

As initial conditions, in model-I, we use two different setups; i) circular and ii) linear. Here, i) and ii) are introduced respectively to focus on a single head formation and the instability at the moving front of avalanche. Simulations using i) show that the formation of a single head and the vortex convection inside the avalanche independent of the particle radius (Fig. 1(a)), whereas an increase in the particle radius enhances an effect to push the rear particles forward. Moreover, it is found that the linear relationship among the head thickness and width rescaled by the particle radius. On the other hand, simulations using ii) show that the air drag destabilizes the initial straight front of avalanche to deform into a wavy pattern with many heads (Fig. 1(b)). In addition, the width of head increases with the number of particles constituting a head and gives a linear relationship with them.

Next, in model-II, we use a triangular shape as initial starter (Fig. 2(a)). Because of using the lightweight particles such as polystyrene, the head-tail structure is formed with time independent of the particle radius (Fig. 2(b)). Furthermore, the particle velocity against the traveling direction of avalanche has peaks inside the avalanche.

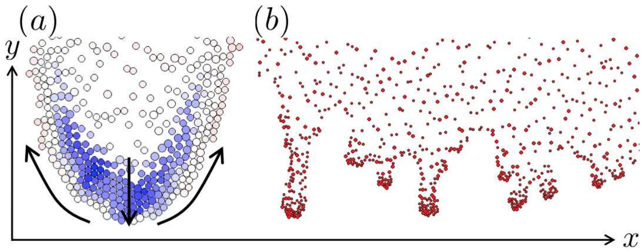

Figure 1. Model-I; (a) particle velocity inside avalanche using circular starter and (b) instability at moving front of avalanche using linear starter.

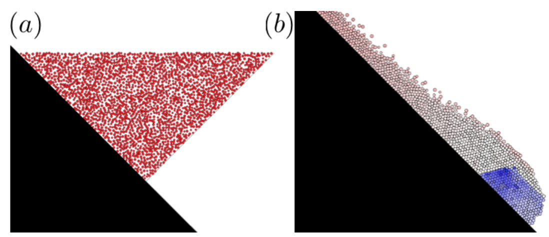

Figure 2. Model-II; (a) initial condition of particles and (b) head-tail structure and particle velocity along slope inside avalanche.

CONCLUSION

In this study, we proposed a particle method with the hydrodynamic interaction using Rotne-Prager tensor and investigate the pattern formation of granular avalanche on two-dimensional slope. These numerical simulations show that the pair of vortexes emerges inside the avalanche and the relation among the particle radius, head thickness, and head width satisfy the scaling low. Moreover, this model represents the wavy pattern and head-tail structure obtained in the experiments consisting of the lightweight particles. These results qualitatively corresponded to the previous experiment.

References

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