HOPPER FLOW PREDICTION USING A CONSTITUTIVE MODEL WITH MICROSTRUCTURE EVOLUTION

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Summary Theoretical predications of granular flow in a conical hopper are presented, which are made by a continuum theory employing a recently-developed constitutive model with microstructure evolution [3]. The model is developed for rate-independent granular materials. The closures for the pressure and the macroscopic friction coefficient are linked to microstructure through evolution equations for coordination number and fabric. The material constants in the model are functions of particle-level properties. A salient predication is the variable stress ratio along the flow direction, in contrast to the constant ratio assumed in some widely-used plasticity theories, but supported by results obtained from discrete element simulations [2]. Another important consequence demonstrated is that the particle-friction effect on the flow behaviour can directly be probed from this continuum calculation. The effect of normal-stress difference on the calculated stress distribution is also examined.

FORMULATION

We consider an incompressible cohesionless material of mass density ρ and volume fraction ϕ discharging from a narrow-angled conical hopper of half angle α in this paper. We further assume radial velocity (v_r) and gravity, zero shear stress, and rigid and smooth walls. The continuity equation is then reduced to

$$v_r = -\frac{V}{r^2}, \tag{1}$$

where V is a positive flow rate independent of r. Utilising the constitutive model in [3], we obtain the dimensionless momentum conservation equation

$$\frac{d\hat{\sigma}_{rr}}{d\hat{r}} = \frac{2\phi\hat{V}^2}{\hat{r}^5} + \frac{2(K-1)\hat{\sigma}_{rr}}{\hat{r}} - \phi, \tag{2}$$

where r is scaled by the outlet radial distance r_0, i.e. $\hat{r} = r/r_0$; and the normal stress $\hat{\sigma}_{rr} = \sigma_{rr}/(\rho g r_0)$ and $\hat{V} = V/\sqrt{r_0^5 g}$. The normall stress ratio $K = \sigma_{\theta\theta}/\sigma_{rr}$ is related to a macroscopic friction η as

$$K = \frac{1 + \frac{1}{\sqrt{3}}(\eta - a_3 A_{rr})}{1 - \frac{2}{\sqrt{3}}(\eta - a_3 A_{rr})}, \tag{3}$$

where a_3 is a material constant dependent on particle friction, A_{rr} is the fabric component and η is related to fabric as

$$\eta = b_1 + \sqrt{3} b_2 A_{rr}, \tag{4}$$

where b_1 and b_2 are material constants and dependent on particle friction. In Eq. 3, the terms containing a_3 arise from the normal stress difference (NSD) during simple shear. The fabric evolution is given as

$$\frac{dA_{rr}}{d\hat{r}} = -\frac{1}{\hat{r}}(2c_1 + \sqrt{3}c_2 A_{rr} + 3c_3 A_{rr}^2), \tag{5}$$

where c_1 to c_3 are material constants (refer to [3] for the numerical values of all the material constants).

RESULTS

Solving equations 2 and 5, the stress and fabric radial distribution are obtained as shown in Fig. 1. The present theory predicts that the normal-stress ratio K varies along the flow direction (see Fig. 1 (a)), which naturally arises from the variation of the fabric (A_{rr} as shown in Fig. 1 (c)). In many previous theories, such as the hour-glass theory (HGT) [1], K is assumed to be a constant. Predictions of normal stress $\hat{\sigma}_{rr}$ from the HGT and from the present formulations with and without the NSD effect are compared in Fig. 1 (b). The constant K is taken as the mean of the K values from the calculation with NSD. The profiles are similar with the HGT result being the largest, which also results in the highest flow rate.

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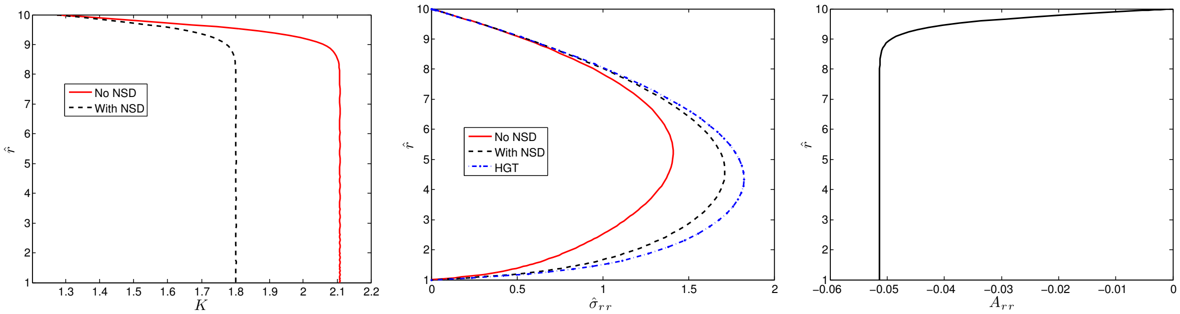

(a) (b) (c)

Figure 1. Radial variation of (a) stress ratio, (b) normal stress and (c) component of the fabric tensor, plotted against the dimensionless distance in the vertical axis. The material has particle friction $\mu = 0.3$.

References

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