Vorticity Banding and Stress Localization in a Granular Fluid*

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In many complex fluids, including granular systems, the homogeneous shear flow is unstable above a critical applied shear rate or shear stress. The flow then separates into coexisting bands of different internal structures and rheological properties. Such shear-induced phenomenon, known as *shear banding* [1,2], occurs in a host of complex fluids: colloidal suspensions, worm-like micelles, soft glasses, foams and emulsions, carbon nano-tube suspensions, liquid-liquid biphasic system, etc. Depending on the directional alignment of banding, they are sub-classified as: *gradient* [1] and *vorticity* [2] banding. When an applied shear rate exceeds a critical shear rate the flow breaks into alternate regions of low and high shear rates, respectively, and this is known as gradient banding. In this case the flow separates into bands of different shear rates (i.e. *shear localization*) along the gradient direction. On the other hand, if the applied shear stress exceeds a critical value, the homogeneous flow separates into bands of different shear stresses (having the same shear rate) along the vorticity (spanwise) direction, leading to *stress localization*. Both gradient and vorticity bandings can partly be explained from the "non-monotonic" behavior of the underlying constitutive curve (the shear-stress versus shear-rate curve under homogeneous deformation); however, the origin of vorticity banding in complex fluids is still under debate [2].

In the realm of granular flows, the gradient banding routinely occurs in earth-bound shear-cell experiments of dense granular materials [3] and has also been found in particle simulations for a range of densities from the dilute to dense regimes [4] Such gradient banding has been explained in terms of the nonlinear saturation of shear-banding instabilities of the underlying hydrodynamic equations via an order-parameter (Landau) equation [5] at least in the dilute-to-moderately dense flows. The particle simulations of three-dimensional granular plane Couette flow [4a] found bandings of particles in the form of streamwise rolls with different mesoscopic structures for dilute flows – this is nothing but 'vorticity-banding' as discussed above. A similar type of vorticity-banding, leading to an ordering transition, has also been observed in simulations of dense granular flows [4b].

What is the origin of vorticity-banding in granular systems? Does it lead to 'stress-localization' as in other complex fluids, or, does it leave an altogether different 'rheological' signature? Since hydrodynamic equations are likely to hold in granular fluids, another key question is: can such 'vorticity-banding' be explained in terms of an order parameter theory derived from granular hydrodynamic equations [5]?

In this paper, we outline a weakly nonlinear order-parameter theory for such vorticity banding in a sheared granular fluid [6]. Our nonlinear analysis holds for any constitutive model, even though we will present results for a kinetic theory constitutive model. The resulting differential equations at any order of perturbation amplitude have been solved analytically, leading to exact calculation of both first and second Landau coefficients. The calculation of higher-order Landau coefficients helped to elucidate degenerate bifurcations that occur at moderate values of particle volume fraction. Our theory predicts that the banding

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of particles into alternate dense and dilute layers, along the mean vorticity direction, can occur via supercritical/subcritical pitchfork (stationary) and subcritical Hopf (oscillatory) bifurcations in dilute and dense flows, respectively, resulting in inhomogeneous states of shear stress (or viscosity) and pressure. The phase diagram for vorticity banding and the scaling of equilibrium amplitudes (in terms of stress inhomogeneity) will be discussed [6].

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