THREE DIMENSIONAL SIMULATION OF GRANULAR JET SCATTERING

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Summary Scattering caused by a three dimensional (3D) incohesive granular jet is investigated numerically. We found the following three properties (i)-(iii) for a cylindrical jet and a jet with rectangular cross section: (i) For a cylindrical case, the bulk behavior can be understood by a momentum balance equation. (ii) The motion of a particle in the boundary region can be treated as the motion of a single particle. (iii) For a jet with rectangular cross section, the scattered fragments tend to flow into the direction with large pressure gradient. This mechanism might be the reason to obtain an elliptic flow.

INTRODUCTION

Let us think of an impact process that some staffs are bumped into a fixed wall. This process can be found in various fields from granular matter to nuclei. If the granular jet flows into the wall continuously, a variety of scattering processes can be observed, though we still do not have any good physical explanation.[1, 2]

The aim of this study is to extract universal properties of the impact processes by investigating the cylindrical granular jet scattering and the jet with rectangular cross section. (i) For the cylindrical case, the bulk behavior can be understood by a momentum balance equation. (ii) The motion of a particle in the boundary region can be treated as the motion of a single particle. (iii) For the jet with rectangular cross section, the scattered fragments tend to flow into the direction with large pressure gradient. This mechanism might be the reason to observe an elliptic flow. We simply use incohesive particles to extract essence of the mechanism. Throughout this study, we adopt the standard Discrete Element Method for soft particles.

SCATTERING OF A CYLINDRICAL GRANULAR JET

We discuss the scattering angle and the collimated ejecta for granular jets. The first one is concerning the scattering angle of the jet, and the second one is concerning whether the ejecta is collimated.

Scattering angle of the granular jet

First, we study the scattering angle ψ_0 of a collimated ejecta in the granular jet for several restitution coefficients e, where ψ_0 strongly depends on $X \equiv D_{\rm tar}/D_{\rm jet}$ with the diameters of the jet $D_{\rm jet}$ and of the target $D_{\rm tar}$. Based on the momentum balance equation, Clanet[3] proposed a phenomenological model of the scattering angle of the water jet without surface tension. Huang *et al.* performed the simulation of the 2D granular jet scattering[2], but they did not study how the results depend on the coefficient of restitution. We verified that the theory by Clanet[3] can be used regardless of the value of the restitution coefficient (Fig.1).

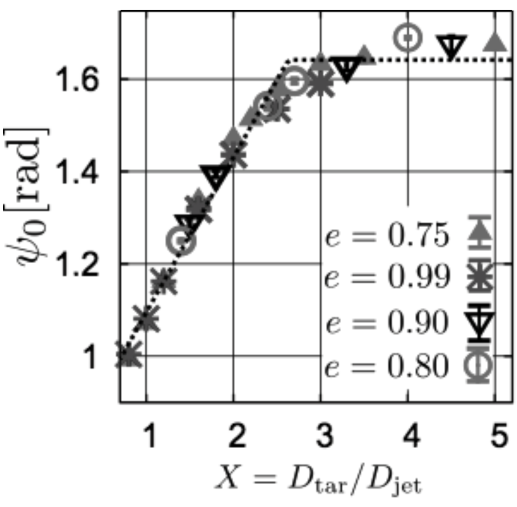

Figure 1. Our 3D numerical result of the scattering angle. The dotted line denotes the prediction from the phenomenological model proposed by Clanet[3].

Collimated ejecta or Firework-like ejecta

It is known that the ejecta is collimated for small $d/D_{\rm jet}$ with the particle diameter d, and is similar to a firework for large $d/D_{\rm jet}$. The ratio $d/D_{\rm jet}$ is related to ratio of the area of the boundary region to that of the bulk of the injected jet. This change of patterns is characterized by the half width $\theta_{\rm half}$ of the angular distribution of the scattered particles. Cheng *et al.*[1] found the relation $\tan\theta_{\rm half} \propto d/D_{\rm jet}$ in their experiments, and we also reproduce the relation from our simulation (Fig.2). It is notable that the relation $\tan\theta_{\rm half} \propto d/D_{\rm jet}$ can be explained by a simple phenomenological theory in which $\tan\theta_{\rm half}$ is estimated to be the ratio of collimated flux $p_\parallel$ to the vertical flux $p_\perp$ as $\tan\theta_{\rm half} \sim p_\perp/p_\parallel$. Note that particles injected from the boundary region collide with fewer particles than those injected from the bulk region do. Then, we may assume that particles from the bulk region whose area is $S_{\rm bulk} \sim \pi D_{\rm jet}^2/4$ contribute to the collimated flux $p_\parallel \sim p_0 S_{\rm bulk}/S$ and those from the boundary region whose area is $S_\partial \sim \pi D_{\rm jet} d$ contribute to $p_\perp \sim p_0 S_\partial/S$, where the area for all scattered particles is S and injected flux is p_0. Therefore we finally obtain

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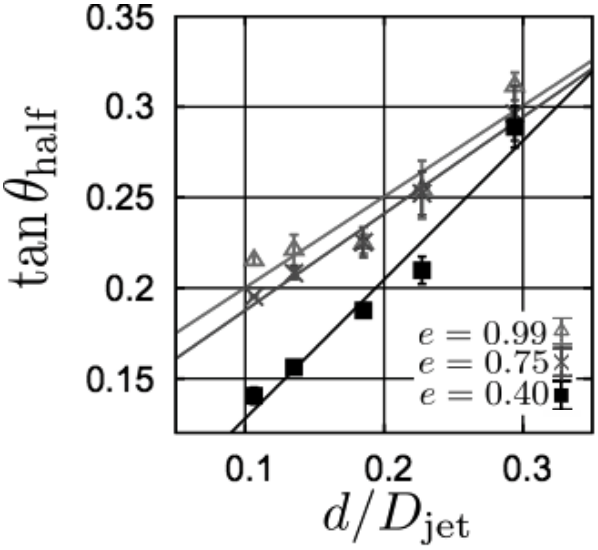

Figure 2. The linear relation between $\tan\theta_{\rm half}$ and $d/D_{\rm jet}$ for several restitution coefficients e.

the relation $\tan\theta_{\rm half} \sim p_\perp/p_\parallel \sim S_\partial/S_{\rm bulk} \propto d/D_{\rm jet}$.

SCATTERING OF GRANULAR JET WITH RECTANGULAR CROSS SECTION

If we change the aspect ratio of the jet, the scattered particles may prefer the certain direction because the pressure gradient is not isotropic around the jet axis. The resulted flow of anisotropic scattering is known as the elliptic flow. The elliptic flow is characterized by the value v_2 which is the coefficient of $\cos 2\varphi$ with the azimuthal angle φ for scattering flux $dN/d\varphi$. It should be noted that the scattering flux $dN/d\varphi$ is related to the differential scattering cross section $d\sigma(\theta,\varphi)/d\Omega$ with the scattering angle θ and the other coefficients v_n with $n = 0, 1, \cdots$ as

$$\frac{dN}{d\varphi} \equiv \int d\cos\theta \frac{d\sigma}{d\Omega} = \sum_{n=0}^{\infty} v_n \cos n\varphi. \tag{1}$$

We prepare the rectangular jet where the length of long and short side are L_y and L_z, respectively. In order to study the geometric dependence of the steady impact process, we change L_y with fixing L_z. We obtain the relation $v_2 \propto L_y/L_z$ (Fig.3), which may result from the anisotropy of the pressure gradient. This mechanism might be the reason that the resultant flow turns out to be the elliptic flow.

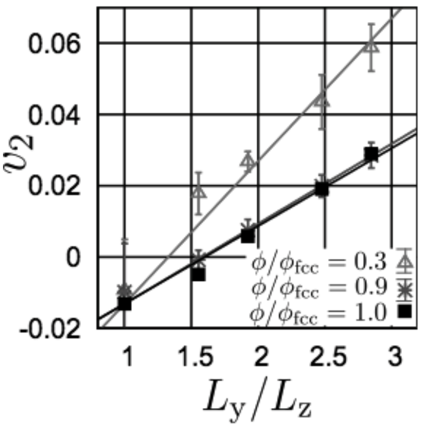

Figure 3. The geometry L_y/L_z dependence of v_2 for several volume fractions, where ϕ and $\phi_{\rm fcc}$ are respectively the volume fraction of the system and the volume fraction of fcc crystal.

CONCLUSIONS

We found that our 3D numerical results of the scattering angles are on the universal theoretical prediction proposed by Clanet[3] regardless of the value of the restitution coefficient. We proposed a simple model to reproduce both a collimated ejecta and a firework-like ejecta. For the granular jet with rectangular cross section, we reproduce the elliptic flow which might be the result from anisotropic pressure gradient.

References

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