OPTIMUM STRUCTURE DESIGN OF A MULTILAYER PIEZO-COMPOSITE DISK FOR CONTROL OF THERMAL STRESS

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Summary This paper deals with a control problem of a thermal stress in a composite circular disk consisting of a structural layer and multiple piezoelectric layers. A new optimization technique applicable to the structure design of the composite disk is developed. Employing the technique, the layer thicknesses, electrode arrangements, and applied voltages are successfully determined in order to maximize the function of stress control. Finally, the maximum thermal stress induced in the structure layer is suppressed by 40.8%.

INTRODUCTION

One of important key technologies of realizing a plan for a hypersonic aircraft is how to develop an adaptive structure member resisting a thermal loading even when it exceeds the allowable limit accidentally. The possibility of controlling a thermal stress in a composite disk with piezoelectric layers was demonstrated, when the structural layer was exposed to a prescribed heating temperature [1]. Voltages applied to electrodes on the piezoelectric layers were determined by optimization in order to minimize the maximum thermal stress in the structural layer. For more successful and efficient control of the maximum thermal stress, the optimum structure design of the composite disk is needed, but the solution space of the optimization problem is multimodal. The authors tried to solve the problem with a random search approach based on the particle swarm optimization (PSO). It was very hard to obtain the optimum solution, because there were many optimization variables in the case of multiple piezoelectric layers with a number of electrodes. In addition, the optimization variables have strong dependence on each other, namely the optimum geometrical variables can be obtained only when the optimum electrical variables are determined accurately. A hybrid optimization technique (HOT) which can determine the electrical variables and the geometrical variables separately is developed. Improvements on HOT are also investigated so that a stable optimum solution is quickly obtained. Finally, the optimum design problem has been solved for a multilayer piezo-composite disk and the performance of stress control is successfully improved.

PRESENTATION OF STRESS CONTROL PROBLEM

Consider a composite disk consisting of a transversely isotropic structural layer $(i=0)$ onto which multiple piezoelectric layers of class 6mm $(1\le i\le N)$ are perfectly bonded, as shown in Fig. 1. It is assumed that a number of electrodes are arranged concentrically on the upper surface of each piezoelectric layer and the lower surface is electrically grounded.

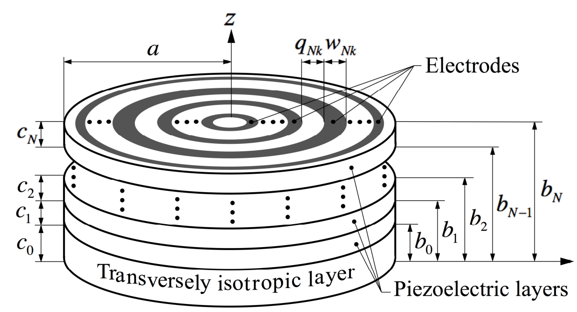

Fig. 1 Geometry of a composite disk

Thermo-elastic response

It is assumed that a heating temperature $T_c f(r)$ acts on the bottom surface of the composite disk:

$$T_{0,z} - h_0 T_0 = -h_0 T_c f(r) \text{ on } z = 0 \tag{1}$$

and heat convection occurs on the top surface. In Eq. (1), T_0 is the temperature change in the structural layer, T_c is a constant temperature, h_0 is the coefficient of relative heat transfer, and $T_{0,z} = \partial T_0 / \partial z$. When the upper surfaces of all piezoelectric layers are free of electric charge, the analytical solution for the temperature change and thermoelastic response in each layer has been obtained by employing the potential function methods. Hereafter, the thermal stresses in the i th layer are expressed by σ_{ijk}^T.

Electro-elastic response

It is assumed that a stepwise voltage is applied to each electrode on the i th piezoelectric layer:

$$\Phi_i^E = \sum_{k=1}^{M_i} V_{ik} \{H(r - r_{ik}) - H(r - r_{ik} - w_{ik})\} \text{ on } z = b_i \quad (i = 1 \sim N) \tag{2}$$

where V_{ik} is a voltage applied to the k th electrode in the i th layer (i.e., $(i-k)$ th electrode), M_i is the number of electrodes on the i th layer, $H(r)$ is Heaviside's unit step function, and r_{ik} is the inner radius of the $(i-k)$ th electrode. Now, let us consider a case where the unit voltage V_u is applied to only the $(n-m)$ th electrode. The discrete response quantities such as the stresses $(\Delta\sigma_{ijk}^E)_{nm}$ resulting from the unit voltage have been obtained by employing the potential function methods and they can be calculated in advance. When a voltage of arbitrary magnitude V_{nm} is applied to every electrode, the electro-elastic stresses in the i th layer are given by the superposition as

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$\sigma_{ijk}^{E} = \sum_{n=1}^{N} \sum_{m=1}^{M_n} P_{nm} (\Delta\sigma_{ijk}^{E})_{nm}$, where $V_{nm} = P_{nm} V_u$ and P_{nm} are the magnification factors.
The resultant stresses induced in the i th layer by both thermal and electric loads are obtained as $\sigma_{ijk} = \sigma_{ijk}^{T} + \sigma_{ijk}^{E}$.

Optimum design problem

Let us solve the optimum structure design problem of determining the thicknesses of the piezoelectric layers c_i , the dimensions of the electrodes w_{ik} and q_{ik} , and the voltages V_{ik} applied to all electrodes so that the maximum thermal stress in the structural layer is minimized subject to constraints on the stresses in the piezoelectric layers. The optimization problem is very difficult to be solved with PSO, because of the dependence between the geometrical and electrical variables. Employing the superposition of the discrete response quantities induced by every unit voltage, the optimization problem of determining the applied voltages V_{ik} can be transformed into the linear programming problem of determining the magnification factors P_{nm} . Therefore, the optimization variables are divided into two categories, in which case one category contains the electrical variables and the other category contains the geometrical variables. The electrical variables are accurately determined by employing the simplex method, whereas the geometrical variables are determined by employing PSO. This method is termed HOT. The optimum design problem is then defined as:

$$\left.\begin{array}{ll} \text{find} & \boldsymbol{X}=(c_1, w_{11}, q_{11}, \bullet\bullet\bullet, c_N, w_{NM_N}, q_{NM_N}),\ \boldsymbol{Y}=(P_{11}, P_{12}, \bullet\bullet\bullet, P_{NM_N}, R_s) \\ \text{to minimize} & f_{obj}(\boldsymbol{X},\boldsymbol{Y}) = R_s \,|\, \sigma_{0\max}^{T} \,| \\ \text{subject to} & \{|\sigma_{0rr}|, |\sigma_{0\theta\theta}|, |\sigma_{0zz}|, |\sigma_{0rz}|\} \le R_s\,|\sigma_{0\max}^{T}|,\ \sigma_{pc}^{A} \le \{\sigma_{prr}, \sigma_{p\theta\theta}, \sigma_{pzz}\} \le \sigma_{pt}^{A},\ |\sigma_{prz}| \le \sigma_{ps}^{A},\ \sum_{k=1}^{M_i} (w_{ik} + q_{ik}) \le a \end{array}\right\} \quad (3)$$

where $R_s = \sigma_{0\max} / \sigma_{0\max}^{T}$, $\sigma_{0\max}^{T}$ and $\sigma_{0\max}$ are the maximum stresses in the structural layer before and after applying the determined voltages, σ_{prr} , $\sigma_{p\theta\theta}$, σ_{pzz} , and σ_{prz} are the maximum stress components in the piezoelectric layers, and σ_{pt}^{A} , σ_{pc}^{A} , and σ_{ps}^{A} are the allowable tensile, compressive, and shear stresses.

NUMERICAL RESULTS

Numerical results are presented in the dimensionless quantities:

$$(\bar{r}, \bar{z}, \bar{c}_i, \bar{b}_i, \bar{q}_{ik}, \bar{w}_{ik}) = (r, z, c_i, b_i, q_{ik}, w_{ik}) / a, \quad \bar{\sigma}_{ijk} = \sigma_{ijk} / (\tilde{\alpha}_r \tilde{Y}_r T_c)$$

where $\tilde{Y}_r$ and $\tilde{\alpha}_r$ are Young's modulus and the coefficient of linear thermal expansion in the structural layer. Numerical calculations have been carried out for a composite disk consisting of a CFRP layer and cadmium selenide layers [1], assuming as follows:

$$f(\bar{r}) = H(\bar{r} - \bar{r}_o)(1 - 2\bar{r}^2/\bar{r}_o^2 + \bar{r}^4/\bar{r}_o^4),\ \bar{r}_o = 0.5,\ B_0 = 1.0,\ B_N = 0.1,$$
$$\bar{c}_0 = 0.002,\ \bar{w}_{ik} \ge 0.025\ \ (1 \le k \le M_i),\ \bar{q}_{ik} \ge 0.025\ \ (2 \le k \le M_i),$$
$$\bar{\sigma}_{pt}^{A} = 0.004,\ \bar{\sigma}_{pc}^{A} = -0.04,\ \bar{\sigma}_{ps}^{A} = 0.002.$$

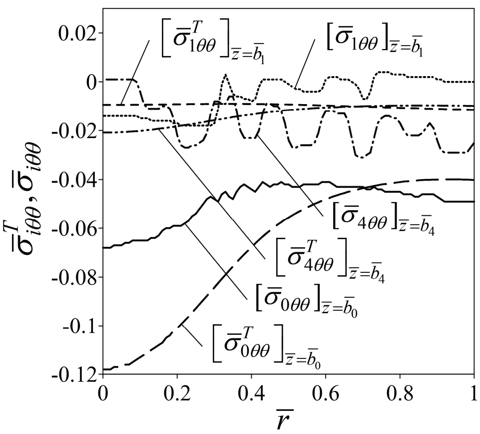

Fig. 2 Hoop thermal stress distributions

As the optimization variables increase, the execution time required for solving the optimization problem (3) and the differences among the optimum solutions obtained for the same conditions increase. In order to resolve the issues, functions of controlling particle motions are added to the PSO algorithm, while the function of selecting active reference points for evaluating the stress constraints is added to the simplex method algorithm. The strategy to efficiently design the multilayer composite disk is also developed. Employing the improved HOT, the execution time is successfully cut down to a fortieth of it in the case of HOT and stable optimum solutions are obtained. Then, the optimization problem (3) has been solved for the cases of two, three, and four piezoelectric layers with a number of electrodes. The maximum thermal stresses are suppressed by 39.7%, 40.7%, and 40.8%, when five electrodes are arranged on the piezoelectric layers except the top layer and seven electrodes are arranged on the top piezoelectric layer. The maximum suppression ratio is considerably higher than that obtained in the authors' previous papers. Figure 2 illustrates the hoop stress distributions on the selected layer surfaces before and after applying the voltages for the case of the composite disk with four piezoelectric layers. The maximum thermal stress yielded at the center of the structural layer surface is reduced from -0.1178 to -0.0697, while the stresses in the piezoelectric layers are increased, but less than the stress constraints.

CONCLUSIONS

The stable optimum solutions to the complicated structure design problem of the multilayer composite disks have been obtained by employing the improved HOT. For the case of this model, it is deduced from the numerical results that a possible highest suppression may be a little more than 41%, even if the piezoelectric layers and electrodes are increased.

References

[1] Ashida, F., Sakata, S., and Matsumoto, K.: Structure Design of a Piezoceramic Composite Disk for Control of Thermal Stress, *Trans. ASME, Journal of Applied Mechanics*, **75**: 061009-1~8, 2008.