ON EQUILIBRIUM DOMAINS IN SUPERELASTIC NITI TUBES —HELIX VERSUS CYLINDER

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Summary This paper studies the energetics of the cylindrical and helical domains in tubes and shows that the total misfit energy depends on two nondimensional length scales: the normalized effective domain length and normalized wall-thickness. We quantify the length scale dependence in the energy of equilibrium cylindrical and helical domains. The energetic preference of each type of domain is predicted and the critical condition for helix→cylinder domain transition is established.

Macroscopic cylinder- and helix-shaped deformation domains were observed in NiTi polycrystalline shape memory alloy tubes during the phase transition under uniaxial quasi-static isothermal stretching. Further experiments showed that the occurrence of cylinder or helix domain and its subsequent isothermal evolution strongly depend on the domain volume, tube wall-thickness and loading history. For the thin-walled tubes (with the wall-thickness to mean radius ratio $t/R < 0.3$) as shown in Fig. 1(a), it is firmly shown that during stretching the initial homogeneous deformation of the austenite phase suffers instability and a helical martensite band forms which inclines about 55° to the loading axis. While for the thick-walled tubes (see Fig. 1(b)) under the same loading condition, the nucleated domain prefers a cylinder shape instead of helix.

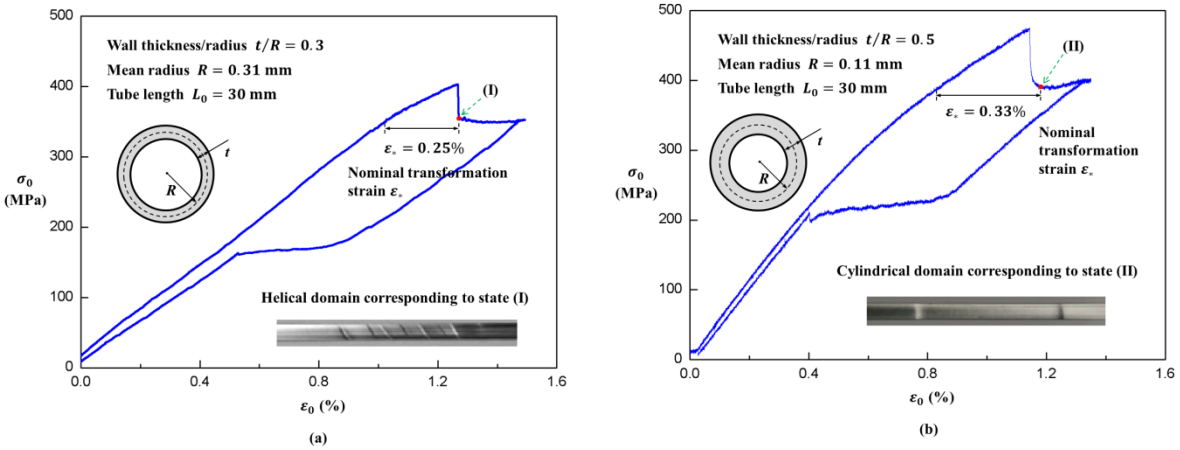

Fig. 1 Typical domain morphologies observed in superelastic NiTi tubes: (a) helical domain in thin-walled tube corresponding to state (I); (b) cylindrical domain in thick-walled tube corresponding to state (II).

Preliminary study on the helical domain evolution in thin-walled tube configurations [1-2] shows that the competition of domain front energy and bending energy of the tube plays important role in the observed isothermal equilibrium domain evolution. For thick-walled tubes, it is expected by intuition that, as the wall-thickness increases, the front energy may become dominant (over the bending energy term) in the total energy of the system and a cylinder domain will be preferred energetically. However, a systematic modeling effort is still needed to quantify the above observed size-dependence of domain morphology and the helix→cylinder morphology transition.

This paper studies the energetics of the cylindrical and helical domains using an elastic inclusion model. A long cylindrical tube ($L_0 \gg R$) with the tube length L_0, uniform mean radius R and wall-thickness t is under consideration (see Fig. 2(a)). The tube contains a cylindrical or helical martensite domain of volume V_Ω (the effective domain length $L_M = V_\Omega/(2\pi Rt)$) with constant axisymmetric (with respect to *z*-axis) eigenstrain ε_{ij}^p (with non-zero components ε_r^p, ε_θ^p and ε_z^p). The elastic constants of the domain and the matrix are assumed to be isotropic and are the same (with Young's modulus E and Poisson's ratio ν). For most shape memory alloys, the volume change in phase transformation is ignorable and these nonzero components can be written as

$$\varepsilon_r^p = \varepsilon_\theta^p = \varepsilon_1^p < 0, \quad \varepsilon_z^p = \varepsilon_2^p = -2\varepsilon_1^p > 0. \tag{1}$$

The normalized total misfit energy $U'_{el}\left(= U_{el}/\left(2\pi Rt \cdot R \cdot E\left(\varepsilon_1^p\right)^2\right)\right)$ of the system consists of two parts,

$$U'_{el} = U'_s + U'_r, \tag{2}$$

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where $U_s'\left(=U_s/\left(2\pi Rt\cdot R\cdot E\left(\varepsilon_1^p\right)^2\right)\right)$ and $U_r'\left(=U_r/\left(2\pi Rt\cdot R\cdot E\left(\varepsilon_1^p\right)^2\right)\right)$ are the normalized bending energy and the normalized front energy, respectively.

The total misfit energy of an equilibrium domain in tube essentially depends on two nondimensional length scales: the normalized effective domain length $L_M'(=L_M/R)$ and the normalized wall-thickness $t'(=t/R)$. For the equilibrium cylindrical domain (where $L_M'>2$, the two fronts are well separated) in tubes with arbitrary wall-thickness/radius ratio (i.e., both thin-walled and thick-walled tubes), the bending energy U_s' and the front energy U_r' can be approximated as

$$U_s'(t')=C_s\cdot(t')^{1/2},\ U_r'(t')=C_r\cdot t', \tag{3}$$

where C_s and C_r are constants.

For the equilibrium helical domain, the bending energy U_s' and the front energy U_r' can be expressed as

$$U_s'(L_M',t')=\frac{1}{2}\left[\frac{C_sC_r^2}{2\pi^2\sin^2\phi}\right]^{1/3}\cdot(L_M')^{2/3}\cdot(t')^{5/6},\ U_r'(L_M',t')=\left[\frac{C_sC_r^2}{2\pi^2\sin^2\phi}\right]^{1/3}\cdot(L_M')^{2/3}\cdot(t')^{5/6}. \tag{4}$$

where $\phi=35°$ is the orientation angle of the helical domain.

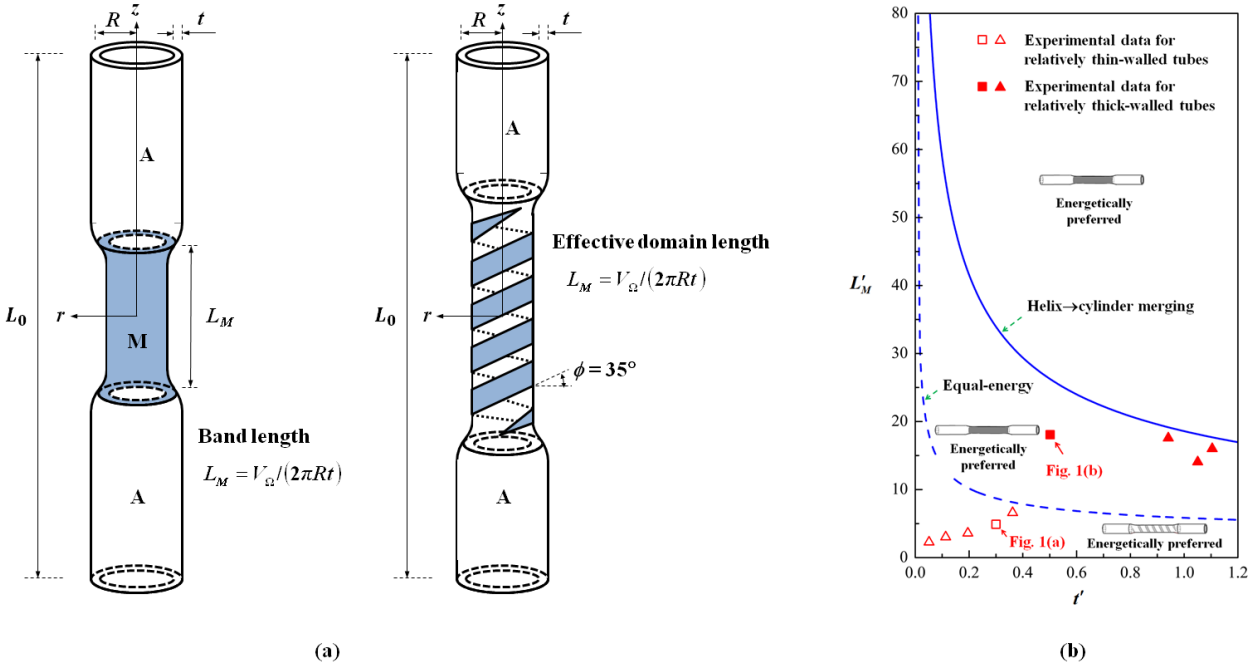

Fig. 2 (a) The cylindrical and helical martensite domains in the tube geometries; (b) Phase diagram showing energetically preferred regions for the cylindrical and helical domains as a function of L_M' and t'.

The energetic preference of equilibrium helical and cylindrical domains in a given tube (t') and at a given effective domain length (L_M') is determined by their energy levels in the tube system. The equal-energy and helix→cylinder merging conditions of the two types of domains are set by two critical functions $(L_M')^0\sim(t')^0$ (dashed line) and $(L_M')^{max}\sim(t')^{max}$ (solid line) in the phase diagram of Fig. 2(b). The energetically preferred regions of the helical and cylindrical domains are given. The results show good agreements with the experiments that the nucleated martensite domain prefers a helical shape for thin-walled tubes (see open symbols in Fig. 2(b)) and prefers a cylinder shape for relatively thick-walled tubes with $t/R>0.5$ (see solid symbols in Fig. 2(b)).

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References

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