EFFECTS OF SURFACE MAXWELL STRESSES ON PIEZOELECTRIC FRACTURE

Cun-Fa Gao[*a)], Haopeng Song[*]
[*] *State Key Laboratory of Mechanics and Control of Mechanical Structures*
Nanjing University of Aeronautics & Astronautics, Nanjing 210016, China

Summary The mode-I crack problem in a piezoelectric solid is re-visited by considering surface Maxwell stresses along crack faces and at infinity of the solid. Using the semi-permeable crack models, the analytical solutions for the electric field inside the crack, the intensity factors of fields and the crack opening are derived analytically based on the complex variable method. Numerical results are also given to illustrate the analytic results, and some new findings are concluded.

DESCRIBTION OF THE PROBLEM

Consider a crack in the piezoelectric solid, as shown in Fig.1. The solid is exposed in air with the electric field E_{env}^{∞} and the crack is filled with gas. In this case, the Maxwell stresses will be induced on the crack surface and also at the outside surface of the solid [1-4].

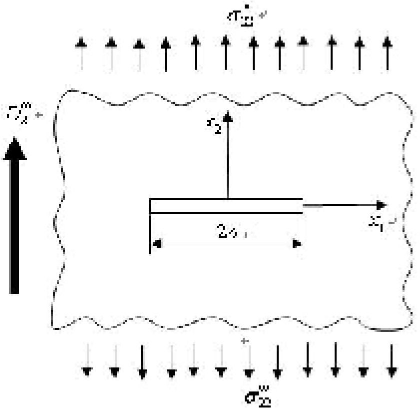

Fig.1. A crack in a piezoelectric solid under combined mechanical-electric loads

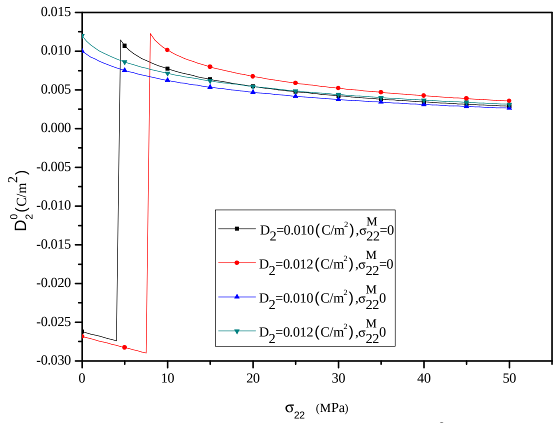

Fig.2 Effects of Maxwell stress on D_2^0 under given electric load

In this case, the electric potential difference across the crack Δf , crack opening Δu_2 and the intensity factors of stresses K are [1]

$$\Delta f = \left[H_{42}\left(s_{22}^{\infty} + \frac{(D_2^{\infty})^2}{2e_v} - \frac{(D_2^0)^2}{2e_v} \right) + H_{44}\left(D_2^{\infty} - D_2^0\right) \right] \sqrt{a^2 - x_1^2}\,, \text{ on crack} \tag{1}$$

$$\Delta u_2 = \left[H_{22}\left(s_{22}^{\infty} + \frac{(D_2^{\infty})^2}{2e_v} - \frac{(D_2^0)^2}{2e_v} \right) + H_{24}\left(D_2^{\infty} - D_2^0\right) \right] \sqrt{a^2 - x_1^2}\,, \text{ on crack} \tag{2}$$

$$K_I = \sqrt{p\,a}\left[s_{22}^{\infty} + \frac{(D_2^{\infty})^2}{2e_v} - \frac{(D_2^0)^2}{2e_v} \right], K_D = \sqrt{p\,a}\left[D_2^{\infty} - D_2^0\right] \tag{3}$$

where $H_{22}, H_{24}, H_{42}, H_{44}$ are the known constants which are determined directly by the material constants of the piezoelectric solids under consideration, K_I and K_D are the intensity factors of stress and electric displacement, respectively, and D_2^0 is the unknown electric field inside the crack, and e_v is the dielectric constant of the environment (air).

SOLUTIONS FOR THE CASE OF A SEMI-PERMEABLE CRACK

In this case, the electric boundary condition on the crack faces can be expressed as [5]

$$D_2^0\left(u_2^+ - u_2^-\right) = -e_v\left(f^+ - f^-\right). \tag{4}$$

Using Eqs. (1)~(4) leads to

$$\left(\Delta D_2^0\right)^3 - 3\left(\Delta D_2^0\right)^2\left[\frac{e_v H_{24}}{H_{22}} + D_2^{\infty}\right] + 2\left(\Delta D_2^0\right)\left[\left(D_2^{\infty}\right)^2 + \frac{2e_v H_{24}}{H_{22}} D_2^{\infty} - e_v s_{22}^{\infty} + \frac{e_n^2 H_{44}}{H_{22}}\right] + 2\left[e_v D_2^{\infty} s_{22}^{\infty} + \frac{e_v^2 H_{24}}{H_{22}} s_{22}^{\infty}\right] = 0, \tag{5}$$

where $H_{42} = H_{24}$ is applied, and $\Delta D_2^0 = D_2^{\infty} - D_2^0$.

a) Corresponding author. E-mail: cfgao@nuaa.edu.cn

Eq. (5) is a cubic equation and its general solution can be found in textbooks. In general, it has three roots, but only one root, which leads to a positive crack opening displacement when a sole mechanical load is applied, is physically acceptable. Hence, once D_2^0 is determined from Eq. (5), the intensity factors of fields can be calculated.

NUMERICAL EXAMPLES

In the following numerical examples, we take $a = 1cm$, $e_v = 8.85\times10^{-12} C^2/N\cdot m^2$, and the model material is PZT-5H with the following Irwin matrix as [1]

$$\begin{bmatrix} H_{22} & H_{24} \\ H_{42} & H_{44} \end{bmatrix} = \begin{bmatrix} 0.803\times10^{-11} m^2/N & 0.639\times10^{-3} m^2/C \\ 0.639\times10^{-3} m^2/C & -2.289\times10^{7} Vm/C \end{bmatrix}.$$

The changes of $D_2^0, \Delta u_{2\max}$ and K_I with the applied mechanical loads under given electric loads are shown in Figures 2-4, respectively, to examine the effects of Maxwell stress at the outside surface of the solid. It can be found from Fig.2 that D_2^0 has a jump as the applied mechanical load increases when the Maxwell stress on the crack faces is considered, but it flatly changes when the Maxwell stress at the outside surface of the solid is also taken into account. Moreover, it can be seen from Fig.3 that if the Maxwell stress is considered only on the crack faces, the initial mechanical load for crack opening is necessary, but it is unnecessary when the Maxwell stress at infinity is also taken into account. Similar is the changes of K_I as shown in Fig.4.

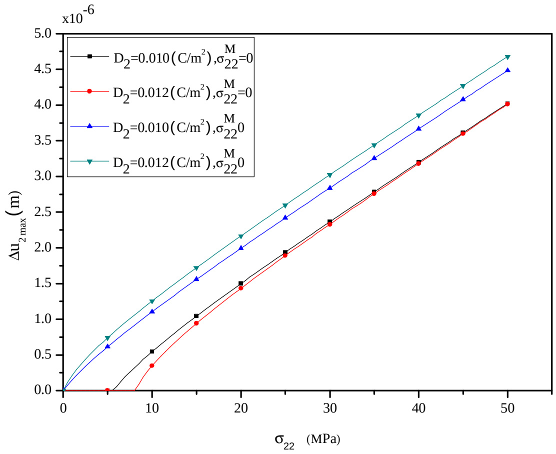

Fig.3 Effects of Maxwell stress on $\Delta u_{2\max}$ under given electric load

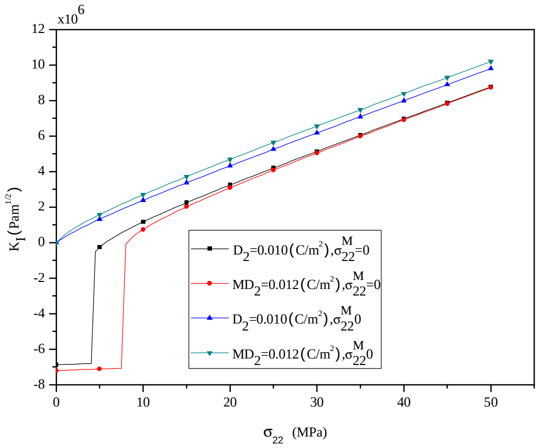

Fig.4 Effects of Maxwell stress on K_I under given electric load

CONCLUSIONS

We revisit the mode-I crack problem in a piezoelectric solid and it is found that when combined mechanical-electric loads are applied, the effect of the surface electrostatic force on piezoelectric fracture may be remarkable. Especially, when the interior of the crack and the surrounding space at infinity are filled with different gases, the Maxwell stress can either enhance or retard crack growth.

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