LIGHT INDUCED CONTRACTION AND BENDING OF LIQUID CRYSTAL ELASTOMERS

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Summary The paper presents a large deflection model for the light-induced bending of liquid crystal elastomers (LCEs). The coupled equations of in-plane force balance and deflection are derived by Hamilton principle. Analytical and numerical solutions indicate that relatively large membrane forces can appear and strongly affect the bending of constrained LCE samples. Their topography can be varied and controlled by changing the spot, the direction, the intensity and the half width of the incoming laser beams.

INTRODUCTION

Light actuations have been reported recently in cross-linked liquid crystal (LC) polymers with photochromic moieties such as Azobenzene. Large contraction[1] and bending[2] can be induced by light illumination with suitable wavelength. The opto-mechanical behaviour is mainly attributed to the photoisomerization of the azo dyes and the subsequent reduction of the order of LC molecules. Thus, an eigenstrain up to 20% (contraction along the LC director) can be produced and a LCE sample can bend rather strongly due to decay of light along propagation. The photochromic LCEs have the potential applications to be remote-controllable actuators and sensors. Several models have been proposed to study the light induced bending of free standing beam or/and plate shaped LCE samples[3-6]. However, the effect of the membrane force will be particularly strong for samples under boundary constraints, which are often encountered in applications. Thus, the present report will focus on the coupled effect of the opto membrane force and moment for constrained LCE samples.

MODEL AND CALCULATIONS

Consider small strain but large deflections, the linear stress-strain relation $\sigma = E\left(\varepsilon^t - \varepsilon^m\right)$ will be considered with ε^m the eigenstrain (opto strain) induced by light through photoisomerization process. By using von Karman strain for the geometric nonlinearity and the Hamilton principle, we can derive the following coupled equations for the membrane force $F_x(t)$ and the deflection $v(x,t)$

$$\frac{F_x(t)}{h} = E_0(t)\left[\varepsilon_0(t) - \varepsilon_0^m(t) + \frac{1}{2L}\int_0^L v_x^{\,2}(x,t)dx - \frac{1}{L}\int_0^L v_{xx}(x,t)\left(\bar{y}(x,t) - \frac{h}{2}\right)dx\right] \tag{1}$$

$$\left(\bar{D}(x,t)v_{xx}(x,t)\right)_{xx} - wF_x(t)v_{xx}(x,t) = -q(x,t) - M_{eff}(x,t)_{xx} \tag{2}$$

Bending under uniform light illuminations

The effective opto moment M_{eff} will be constant along the sample for uniform light illuminations. Eq.(2) can be integrated as $v = c_1 \sinh \lambda x + c_2 \cosh \lambda x + c_3 x + c_4$ with the constants determined by boundary conditions. Then, the membrane force $F_x(t)$ can be obtained by (1). Fig. 1a) shows that $F_x(t)$ increases to approach some steady values upon light illumination for a both-end clamped beam. When the beam is simply supported with fixed length, the effect of membrane force on the bending deflection is obvious as shown in Fig. 1b), where “small” are obtained from small deflection beam model[6]. When the beam is clamped at one end and attached to a spring at the other, the deflection can be adjusted by the spring constants k as shown in Fig. 1 c). When $k/3\mu w = 1e-3$ is very small, the spring has little effect, so the LCE sample bends downward and almost all the deflection is negative under the upward illumination. With the increase of k, the constraint of the spring becomes larger, the negative deflection of the right end becomes smaller, and the deflection of the middle part starts to be positive. When k is very large, such $k/3\mu w = 0.5$ or 10, the deflection of the right end is almost zero. The beam contraction is affected strongly by the spring constant as well.

Beam shapes controlled by laser light

The shape of the LCE beam can be remotely controlled by laser light whose intensity is given by a Gaussian distribution: $I_0(x) = I_m \exp\left(-2\left(x-b\right)^2 / a^2\right)$ with b the illumination center， a the half width and I_m the intensity at x=b.

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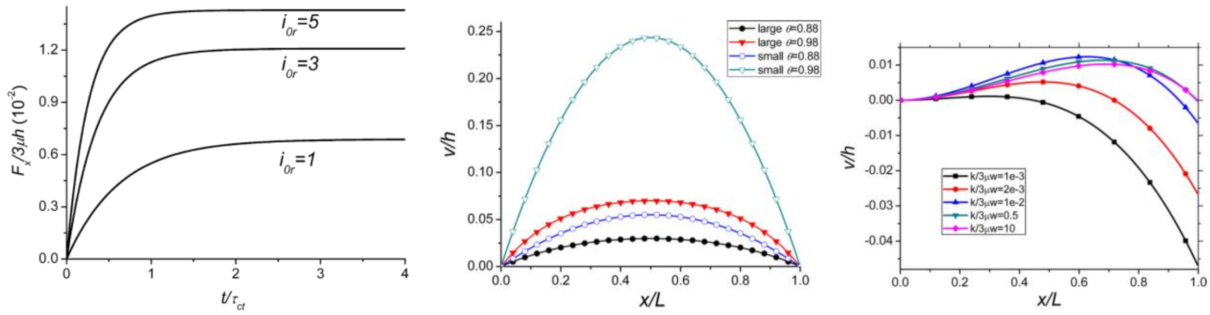

Fig. 1. Beams under uniform light illumination: a) membrane force with two-end clamped; b) deflection curves with two-end simply supported and length fixed; c) deflection curves with left clamped and right attached to a spring.

Fig. 2 a) depicts the effect of the illumination center *b* on the deflection of a two-end clamped beam. It is very interesting to observe that when *b* moves from the middle to the left, first, the position of the largest deflection moves to the left and the magnitude decreases. However, when b is smaller than a critical position, the deflection starts to become negative, due to the constraint of the boundary. When $b = 0.1L$, almost all the deflection curve becomes negative. More sophisticated shapes (patterns) can be obtained by patterned laser light[4] as shown in Fig. 2 b) and c). Thus, predesigned shapes can be obtained by suitable setting of light illuminations.

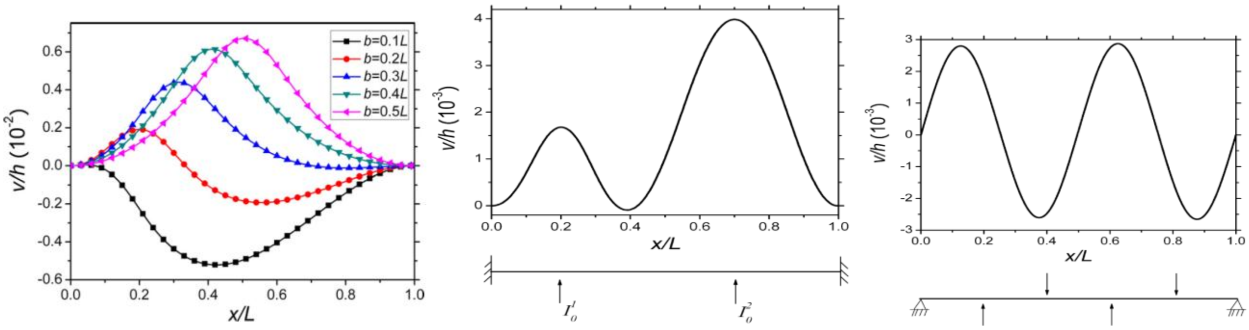

Fig. 2. Various bending shapes under laser light: a) effect of illumination center *b*; b) two-end clamped beam illuminated at two positions; c) two-end simply supported beam illuminated from top and bottom at four positions.

CONCLUSIONS

Photochromic LCEs can be actuated remotely by light with suitable wavelength due to light induced eigenstrains along the LC director. For a beam shaped specimen constrained at two ends, a large membrane force and an opto moment will be produced simultaneously. Von Karman type large deflection model is derived by using the Hamilton principle and should be used to calculate the coupled effect of the light induced contraction and bending of constrained beams. The results show clearly that the membrane force has large effect on the bending and the small deflection theory is not applicable. For the situation of the uniform illumination, we solve the deflection curve differential equation by an analytical-numerical method, and obtain the deflection curves under the clamped, simply supported and spring boundary conditions. As a result, the dependence of the deflection on the light intensity is non-monotonic. Under nonuniform laser light, by changing the illumination position, the illumination direction, the light intensity and the half width of the electric field, we can vary the deflection of the LCE sample and control its shape. For the two-end clamped boundary condition, if we move the laser illumination from the middle to the end, the sample will change its shape from bending away from the laser to toward the laser. Sophisticated patterns can be obtained by patterned laser light illuminations.

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