Dynamic characteristics of rigid cylindrical container with multiple elastic annular baffles and partially filled with liquid

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Summary Coupled dynamic characteristics of multiple elastic annular baffles with liquid partially filled in rigid cylindrical container have been studied. The liquid domain is divided into several simple sub-domains so that the liquid velocity potential in each liquid sub-domain is of class C^1 with continuous boundary conditions. The coupled natural frequencies and corresponding modes have been obtained. For the lateral excitation, the total liquid velocity potential is taken as the sum of the container potential function and the liquid perturbed potential function. Based on the modal superposition principle, the dynamic response of the system is solved.

INTRODUCTION

Free surface of liquid in a partially filled container allows liquid sloshing relative to the container. Liquid sloshing is generally caused by excitation to the container. Under the external excitation, it is significant to determine the hydrodynamic loads acting on the container and the free surface wave height [1]. The performance and stability are the primary concerns in the design of liquid containers. Several passive devices have been adopted in practice to control the liquid sloshing. A common technique is to place baffles in the container to control the liquid sloshing [2]. Therefore, it is important to understand the vibration characteristics of liquid-container interaction where the elasticity of the baffles should be considered in the analysis. Cheung and Zhou [3] and Zhou and Cheung [4] analyzed, respectively, the hydro-elastic vibration of a rectangular container bottom plate and a vertical rectangular plate in contact with water on one side. In the present analysis, the coupled dynamic characteristics of multiple flexural annular baffles with the same inner radius and liquid partially filled in a cylindrical rigid tank are studied.

PHYSICAL DESCRIPTIONS AND MODELING

Consider a rigid cylindrical container with multiple thin elastic annular baffles, which is partially filled with inviscid, incompressible and irrotational liquid, as shown in Figure 1. The effect of free surface waves are considered in the analysis, and a linear theory of liquid movement is adopted from the small-amplitude sloshing of the liquid. In the present study, a two-step partition is developed in the solution process, as shown in Figure 2. The first-step partition is to divide the liquid domain into cylindrical domains between baffles. The second-step partition is to divide each cylindrical liquid domain into a cylindrical sub-domain and an annular cylindrical sub-domain. By means of such a partition, the liquid velocity potential in each sub-domain is of class C^1 with continuous boundary conditions.

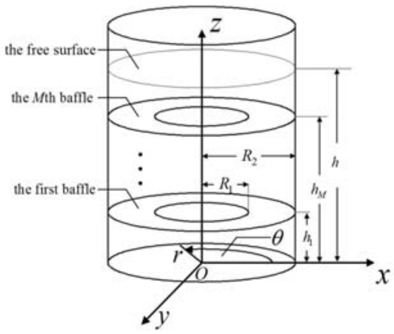

Figure 1 Cylindrical container with multiple annular rigid baffles

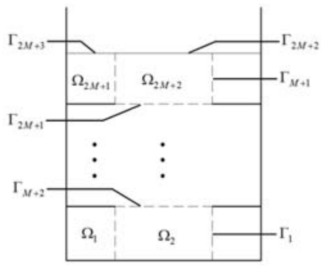

Figure 2 Sub-domains and interfaces

According to the above assumptions and definitions, the velocity potential ϕ_i of liquid movement should satisfy the Laplace equation:

$$\nabla^2\phi_i = 0, \ (i = 1, 2, ..., 2M + 2) \tag{1}$$

The governing equation of the *p*th baffle under the hydrodynamic pressure distribution is

$$D_p\nabla^4 w_p + \rho_p t_p \ddot{W}_p = \rho_l \left(\dot{\phi}_{2p+1}\Big|_{z=h_p} - \dot{\phi}_{2p-1}\Big|_{z=h_p} \right), \ (p = 1, 2, ..., M) \tag{2}$$

in which, $D_p = E_p t_p^3 \Big/ [12(1-\nu_p^2)]$ denotes the bending stiffness of the baffle, w_p denotes the dynamic deflection of the baffle, t_p denotes the thickness of the baffle.

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MODAL ANALYSIS

When liquid undergoes free sloshing in the container, the velocity potential are the harmonic functions of time, i.e. $\phi_i(r,\theta,z,t) = j\omega e^{j\omega t}\Phi_i\left(r,\theta,z\right)$. For each liquid sub-domain, the mode shapes must satisfy all boundary conditions of the liquid sub-domain. The boundaries of each liquid sub-domain can be separated into two types: homogeneous boundaries and non-homogeneous boundaries. Based on the superposition principle, the liquid mode shape corresponding to sub-domain Ω_i can be written as $\Phi_i = \sum \Phi_i^q$ where Φ_i^q is the component of the mode shape solution which can be solved by the method of separation of variables. The free surface condition, the interface conditions and the coupled vibration conditions are expressed in terms of Fourier series along the liquid height and Bessel series in the radial direction, respectively. The coupled natural frequencies and corresponding modes have been obtained by solving the generalized eigen-value problem. In the following numerical example, the number of the baffles is taken as M=2. When two baffles are positioned at h_1/R_2=0.6 and h_2/R_2=0.8 respectively, the non-dimensional frequency parameters with respect to the inner-outer radius ratio of the baffles is depicted in Figure 3. When the inner-outer radius ratio of the baffles is fixed at R_1/R_2=0.5 and the lower baffle is positioned at h_1/R_2=0.4, the non-dimensional frequency parameters with respect the position ratio of the upper baffle is depicted in Figure 4.

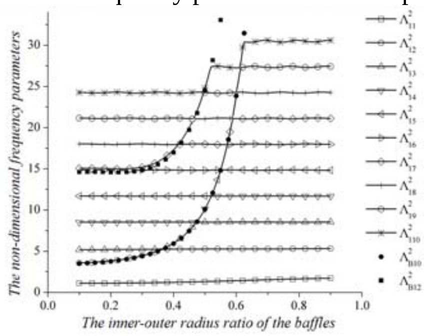

Figure 3 Non-dimensional frequency parameters for two baffles versus the inner-outer radius ratio of the baffles

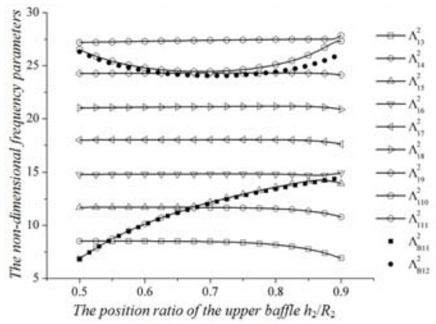

Figure 4 Non-dimensional frequency parameters for two baffles versus the position ratio of the upper baffle

DYNAMIC RESPONSE TO HARMONIC LATERAL EXCITATION

The potential function of each liquid sub-domain can be taken as the sum of liquid perturbed potential function and container potential function. The container potential function is a harmonic function which satisfies the Laplace equation and the relevant boundary conditions. The generalized coordinates are introduced to express the liquid perturbed potential function. The dynamic response equations are established by substituting the total liquid velocity potential into the free surface wave equation and the baffle-liquid coupled equation. The dynamic deflections of the baffles when the static container is suddenly subjected to harmonic lateral excitation are given in Figure 5. The surface wave height at container wall is given in Figure 6.

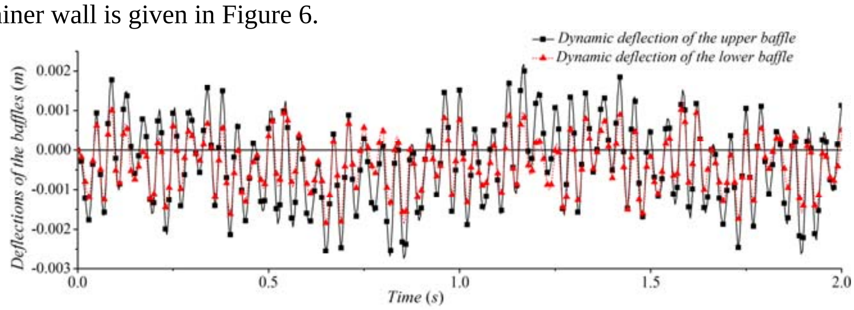

Figure 5 Dynamic deflections of the baffles under harmonic lateral excitation within the first 2 seconds

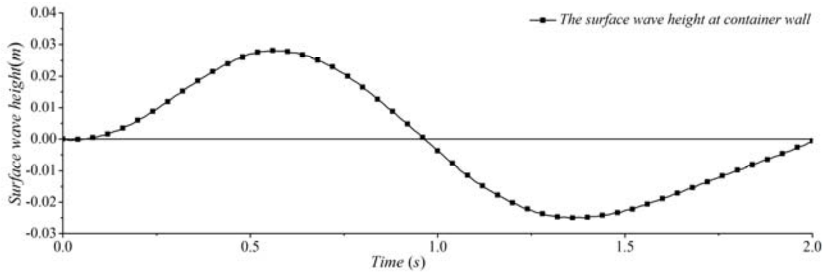

Figure 6 The surface wave height at container wall under harmonic lateral excitation within the first 2 seconds

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