SPH SIMULATION OF FREE SURFACE FLOWS WITH MOVING OBJECTS

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Summary Free surface flows with moving objects are very difficult to simulate for conventional numerical methods as the flows involve involves rapid movement of solid objects, changing and breakup of free surfaces, strong turbulence and vortex, and violent fluid-solid interaction. This paper presented a smoothed particle hydrodynamics (SPH) method for free surface flows with moving objects. Numerical examples demonstrated the effectiveness of the particle approach.

INTRODUCTION

Free surface flows with moving solid objects can be frequently observed in daily life and engineering and sciences, and can be of utmost importance both in theory and practice. Typical examples include landing of aircrafts onto water, diving of sportsman, movement of ships and floating platforms on the sea, and sloshing of liquid containers due to external excitations. Numerical simulation of free surface flows with moving objects is a formidable task and involves rapid movement of solid objects, changing and breakup of free surfaces, strong turbulence and vortex, and violent fluid-solid interaction. Conventional Lagrangian grid based methods such as FEM is difficult in treating large flow deformations, while conventional Eulerian grid-based methods such as FVM and FEM need special algorithms to track or capture the moving free surfaces and moving objects, which usually complicate the problem.

Smoothed particle hydrodynamics (SPH) is a popular meshfree, Lagrangian, particle method with some attractive features [1-3]. SPH uses a set of freely moving particles to represent the state of a system and to approximate the governing partial differential equations, and therefore it can naturally treat moving features and does not need any interface/surface capture or tracking algorithm.

SPH EQUATIONS OF MOTION

In this paper, we present an SPH model for simulating free surface flows with moving objects. Fluid particles are used to model the fluid flows which are governed by Navier-Stokes equations, and solid particles are used to model the dynamic movement (translation and rotation) of moving rigid objects. The interaction of the neighbouring fluid and solid particles renders the fluid-solid interaction, and the non-slip solid boundary conditions. The SPH method is improved with correction on the SPH kernel and kernel gradients, enhancement of solid boundary condition, and implementation of RANS turbulence model.

The Lagrangian form of the N-S (Navier-Stokes) equations are written as follows

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\boldsymbol{u} \tag{1}$$

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{g} - \frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2\boldsymbol{u} \tag{2}$$

where ρ is fluid density, $\boldsymbol{u}$ is the velocity vector, p is pressure, μ is the dynamic viscosity, and $\boldsymbol{g}$ denotes the body force including the gravitational acceleration. The Lagrangian form of N-S equations can be approximated by using SPH particle approximation and the related equations of motion can be obtained [2] as

$$\frac{d\rho_a}{dt} = \sum_b m_b \boldsymbol{u}_{ab}\cdot\nabla_a W_{ab} \tag{3}$$

$$\frac{d\boldsymbol{u}_a}{dt} = \boldsymbol{g}_a - \sum_b m_b\left(\frac{P_a}{\rho_a^2}+\frac{P_b}{\rho_b^2}\right)\cdot\nabla_a W_{ab} + \sum_b \frac{m_b(\mu_a+\mu_b)\boldsymbol{r}_{ab}\cdot\nabla_a W_{ab}}{\rho_a\rho_b(r_{ab}^2+0.01h^2)}\boldsymbol{u}_{ab} \tag{4}$$

where a and b are two interacting SPH particles, $\boldsymbol{u}_{ab} = \boldsymbol{u}_a - \boldsymbol{u}_b$, r is the distance between a and b, and $\boldsymbol{r}$ is the corresponding position vector. m is the particle mass, h is the smoothing length, W is a smoothing function, and $\nabla_a W_{ab}$ is the correspond derivatives over particle a.

For a moving rigid body, the equation for the motion of the centre of mass can be written as follows

$$\frac{d\boldsymbol{u}_o}{dt} = \frac{\boldsymbol{F}}{M} + \boldsymbol{g} \tag{5}$$

where $\boldsymbol{u}_o$ is the velocity of the centre of mass, $\boldsymbol{F}$ is the summation of forces without gravity around the rigid body, M is the mass of the rigid body. In the case of two dimensional motion, the equation for the angular velocity is

$$\frac{d\omega}{dt} = \frac{\boldsymbol{J}}{I} \tag{6}$$

where ω is the angular velocity, I is the moment of inertia, and $\boldsymbol{J}$ is the total moment of force on the rigid body about the centre of mass. The velocity of a point a on the rigid body is

$$\boldsymbol{u}_a = \boldsymbol{u}_o + \omega \times \boldsymbol{r}_{oa} \tag{7}$$

where $\boldsymbol{r}_{oa}$ is the vector from the centre of mass o to point a.

In SPH method, $\boldsymbol{F}$ and $\boldsymbol{J}$ can be expressed as follows

$$\boldsymbol{F} = \sum_b \boldsymbol{f}_b \tag{8}$$

$$\boldsymbol{J} = \sum_b \boldsymbol{r}_{ob} \times \boldsymbol{f}_b \tag{9}$$

where $\boldsymbol{f}_b$ denotes the force on particle b due to fluids. The summation is over all rigid body particles.

NUMERICAL EXAMPLES

Two numerical examples have been modeled to demonstrate the effectiveness of the SPH model for incompressible viscous fluids with moving solid objects. The first one is water entry of a cylinder, and the second example is the movement of a partially-filled rigid box in a wave tank. Fig. 1 and Fig. 2 show the corresponding movements of the free moving objects and the free surface flows at a typical instant. It is demonstrated that the SPH model can well capture the inherent fluid-solid interaction physics with smooth pressure field obtained.

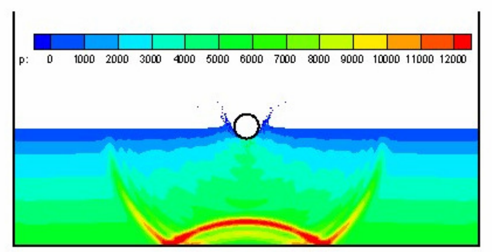

Figure 1 Water entry of a free falling sphere.

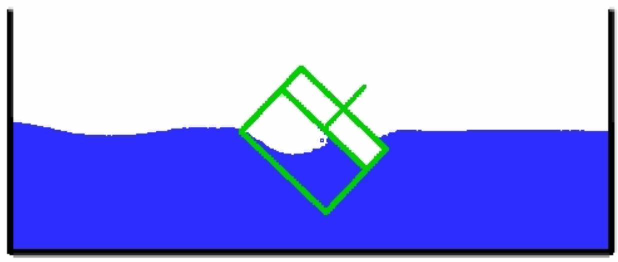

Figure 2 Movement of a partially-filled box in a wave tank.

CONCLUSIONS

As a meshfree, Lagrangian particle methods, SPH does not need algorithms to track or treat free surfaces, deformable interfaces, and moving objects. This avoids many numerical difficulties which conventional grid-based numerical methods are usually fraught with. Therefore the SPH method is attractive in modelling free surface flows with moving objects. Two numerical examples, water entry of a cylinder, and the movement of a partially-filled rigid box in a wave tank, have been simulated. It is shown that the SPH method can well capture the inherent fluid-solid interaction physics with smooth pressure field obtained.

References

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