WAVE INTERACTION WITH TWO ELASTICALLY MOUNTED VERTICAL CYLINDERS

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Summary This paper has examined the interaction of regular waves with two elastically mounted vertical cylinders. The multi-pole expansion method has been employed to solve the problem numerically. Experiments have also been conducted in a wave tank to study the responses of the two cylinders. Comparisons between computed and measured results confirm the accuracy of the prediction method.

INTRODUCTION

With an increase of the application of large cylindrical structures in the sea, the hydrodynamic interaction between water waves and cylinder arrays has received extensive attention[1-3]. The oscillation of elastically mounted cylinders in water waves has been simulated and experimented by many researchers[4]. The emphasis of previous study has largely been on the estimation of the hydrodynamic forces. However, for a large elastically mounted structure, the interaction between wave and structure is significant. This paper presents data for the responses of two flexible, vertical cylinders obtained both theoretically and experimentally.

EIGENFUNCTION EXPANSION METHOD

The geometry of this problem is shown in Fig. 1.The fluid is assumed to be inviscid and incompressible. The motion of the fluid is assumed to be irrotational and can be described by velocity potentials. Under the assumptions of linear water wave theory, the complex velocity potential $\phi(r,\theta,z)$ can be decomposed into two components, the velocity potential due to diffraction of an incident wave acting on a fixed vertical cylinder, ϕ_D, and the velocity potential of wave radiation due to the movement of the elastically mounted vertical cylinder, ϕ_R.

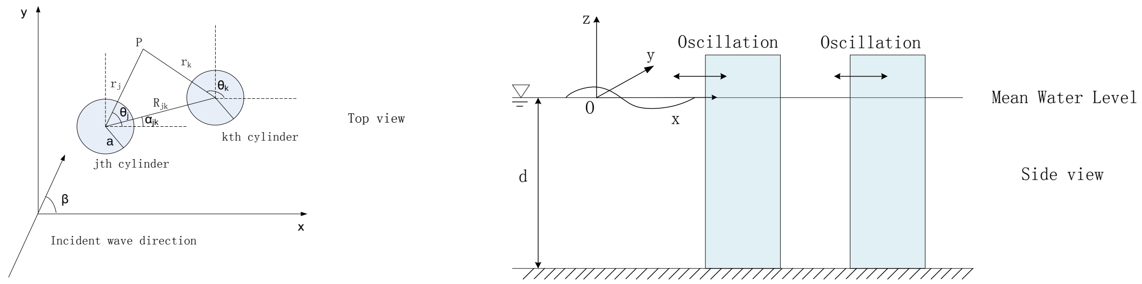

Fig. 1 Definition sketch

In order to solve the interaction problem, the multipole expansion method is employed to give a closed solution. First, the velocity potential due to each body in cylindrical coordinates is expanded into harmonics with unknown coefficients. The Graf's addition theorem is then used to transform to the local coordinates of the other cylinders. The generalized boundary problems for each of the other cylinders are then applied to solve the unknown coefficients.

Diffraction problem

The general form of the velocity potential of the diffraction wave can be written as[2]:

$$\phi_D = \phi_I + \sum_{j=1}^{2} \phi_s^j = e^{i\kappa r\cos(\theta-\beta)} + \sum_{j=1}^{2}\sum_{n=-\infty}^{\infty} A_n^j Z_n^j H_n(\kappa r_j) e^{in\theta_j} \tag{1}$$

The unknown constants will be determined by the following equations:

$$A_m^k + \sum_{\substack{j=1 \\ j\neq k}}^{N}\sum_{n=-M}^{M} A_n^j Z_n^j e^{i(n-m)\alpha_{jk}} H_{n-m}(\kappa R_{jk}) = -I_k e^{im(\pi/2-\beta)} \qquad k=1,\ldots,N, -M<m<M \tag{2}$$

Radiation problem

The general form of the velocity potential of the radiation wave can be written as:

$$\phi_R = \sum_{m=-\infty}^{\infty} [B_{m0}^k H_m(\kappa_0 r_k) Z_0(\kappa_0 z) + \sum_{l=1}^{\infty} B_{ml}^k K_m(\kappa_l r_k) Z_l(\kappa_l z)] e^{im\theta_k} +$$

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$$\sum_{\substack{j=1\\ \neq k}}^{2}\sum_{m=-\infty}^{\infty}[B^j_{m0}Z_0(\kappa_0 z)\sum_{n=-\infty}^{\infty}H_{n+m}(\kappa_0R_{jk})J_n(\kappa_0 r_k)e^{i(m\alpha_{jk}+n\alpha_{kj})}e^{-in\theta_k}+\sum_{l=1}^{\infty}B^j_{ml}Z_l(\kappa_l z)\sum_{n=-\infty}^{\infty}K_{n+m}(\kappa_lR_{jk})I_n(\kappa_l r_k)e^{i(m\alpha_{jk}+n\alpha_{kj})}e^{-in\theta_k}] \quad (3)$$

The unknown coefficients for $l>0$ can be obtained from:

$$B^k_{nl}H'_n(\kappa_l a)+\sum_{\substack{j=1\\ \neq k}}^{2}\sum_{m=-\infty}^{\infty}e^{i(m\alpha_{jk}-n\alpha_{kj})}B^j_{ml}I'_n(\kappa_l a)K_{m-n}(\kappa_0R_{jk})=\begin{cases}\dfrac{X_k\int_{-H}^{0}Z_l(\kappa_l z)\,\mathrm{d}z}{2\kappa_l\int_{-H}^{0}Z_l^2(\kappa_l z)\,\mathrm{d}z}, & n=\pm1\\ 0, & n\neq\pm1\end{cases} \quad (4)$$

Wave induced vibration

The equation used to describe the in-line response of two oscillating vertical circular cylinder in waves can be written as follows

$$(-m_i\omega_i^2-c_i\omega_i+k_i)X_i-F^R_{ix}=F^D_{ix} \quad i=1,2 \quad (5)$$

where m_i is the mass of cylinder in air , c_i is the structural damping of cylinder system in air, k_i is the effective spring stiffness , X_i is the cylinder displacement to be solved, and F^D_{ix} and F^R_{ix} are in-line diffraction force and radiation force on the ith cylinder, respectively to be expressed by the velocity potentials using the linearised Bernoulli's equation.

LABORATORY EXPERIMENTS

To investigate the response of the two elastically mounted vertical cylinders in waves, laboratory experiments were conducted in a wave flume with a piston type wave generator installed at one end and an absorber system installed at the other end to minimize wave reflections as shown in Fig 2. The two cylinders used in the study of the in-line response are made from a plastic tube with an outside diameter of 215mm and a length of 1000mm. It is positioned vertically, with its bottom at a distance 600mm below the water surface and mounted on two steel plates. The structural damping, c, the spring stiffness, k, and the natural frequency of each cylinder are first determined by allowing the cylinder to oscillate freely in air. The experiments were performed using an incident wave height of 10mm and wave periods ranging from 0.5s to 2.0s. The acceleration responses are measured as shown in Fig3.

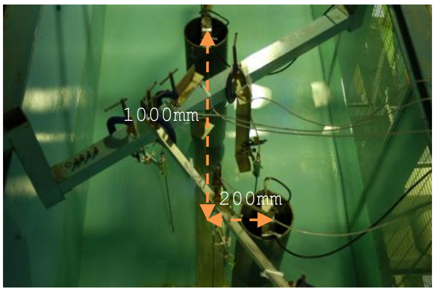

Fig. 2 Two cylinders case

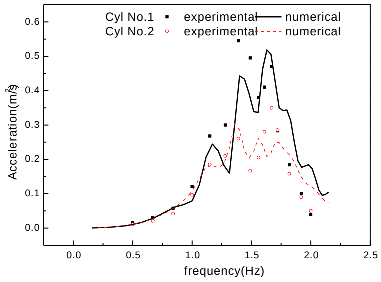

Fig. 3 Comparison between theory and experiment

RESULTS AND DISCUSSION

It is observed that the computed values agree well with the experimental results. The cylinders primarily oscillate at the same frequency as resonance occurs. As can be seen, the response curves differ significantly from that of the single cylinder case. While the single cylinder has only one peak at its natural frequency, it is not the case for the two cylinders. This is due to the interaction between the diffraction and radiation waves from each of the two cylinders.

CONCLUSIONS

This paper has examined the interaction of waves with two elastically mounted cylinders. The interaction problem is separated into conventional hydrodynamic and structural oscillation problems. Good agreement between computed and experimental values is found. The response frequency curves of the two cylinders have more than one peak because of interaction between their oscillations.

ACKNOWLEDGEMENTS

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References

[1] Linton, CM and Evans, DV: The Interaction of Waves With Arrays of Vertical Circular-cylinders. *J. Fluid Mech* **215**:549-569, 1990.
[2] Williams AN, Li W: Water Wave Interaction with an Array of Bottom-mounted Surface-piercing Porous Cylinders. Ocean Eng.27:841-866, 2000.
[3] Walker D, Taylor RE: Wave Diffraction from Linear Arrays of Cylinders. Ocean Eng. 32:2053-2078, 2005.
[4] Utsunomiya T, Taylor E: Resonances in Wave Diffraction/radiation for Arrays of Elastically Connected Cylinders. *J. Fluid and Structures* 14:1035-1051, 2000.