OPTICAL-FORCE-INDUCED MIGRATION OF AN ELASTIC CAPSULE IN A UNIFORM FLOW

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Summary Lateral migration of an elastic capsule by optical force in a uniform flow is studied by numerical analysis. To simulate the coupled system, the penalty immersed boundary method is adopted for fluid-capsule interaction, and the dynamic ray tracing is used for optical force calculation. Effects of the elastic properties of the capsule, i.e. surface Young's modulus and bending modulus, are studied. It is found that the less deformed capsule is more migrated.

Behavior of a liquid-filled elastic capsule by an external force such as hydrodynamic force and optical force has been widely studied in the cellular biology and bioengineering. Typical examples of the capsule are a red blood cell (RBC) and a synthetic capsule for drug delivery. Deformability of a cell is an important indicator in detection and diagnosis of the disease such as malaria and cancer, since its elastic properties are changed significantly under such situations [1]. Continuous separation of rigid particles or biological cells in fluid flow has been performed by optical chromatography and cross type optical particle separation [2, 3]. Hence, for possible applications of efficient separation of abnormal cells, it is important to understand the behavior of an elastic capsule subjected to optical force in fluid flow.

In the present study, we simulate the motion of an elastic capsule subjected to optical force induced by laser beam with Gaussian distribution in a uniform flow as shown in Fig. 1. To solve this problem, fluid-flexible body interaction has to be considered in conjunction with optical force calculation. For the fluid-flexible interaction, we adopt the penalty immersed boundary method (pIBM) proposed by Huang et al. [4]. Governing equations of the fluid flow and the membrane motion are solved separately by using different methods in the framework of pIBM. The fluid motion is governed by Navier-Stokes equation and continuity equation as

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{u} + \boldsymbol{f}, \qquad \nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

where $\boldsymbol{u}$ is the fluid velocity, p is the pressure, Re is the Reynolds number, $\boldsymbol{f}$ is the momentum forcing to apply no-slip condition near the immersed boundary. The fluid flow is defined on the Eulerian domain using a Cartesian coordinate. Unlike the fluid flow, motion of the capsule is described by the Lagrangian variable defined on the moving curvilinear coordinates (s_1,s_2), which is governed by

$$\rho\frac{\partial^2 \boldsymbol{X}}{\partial t^2} + \lambda\frac{\partial \boldsymbol{X}}{\partial t} = \sum_{i,j=1}^{2}\left[\frac{\partial}{\partial s_i}\left(S_{ij}\frac{\partial \boldsymbol{X}}{\partial s_j}\right) - \frac{\partial}{\partial s_i \partial s_j}\left(B_{ij}\frac{\partial^2 \boldsymbol{X}}{\partial s_i \partial s_j}\right)\right] - \boldsymbol{F} + \boldsymbol{F_o} \tag{2}$$

where ρ is the density difference between the solid and fluid, λ is the damping coefficient, S_{ij} is the second Piola-Kirchhoff stress tensor, B_{ij} is the bending stress tensor, $\boldsymbol{F}$ is the Lagrangian momentum forcing representing the fluid-solid interaction, and $\boldsymbol{F}_{\mathrm{o}}$ is the optical force applied on the solid. S_{ij} and B_{ij} are derived using the principle of virtual work for neo-Hookean membrane energy density and bending energy density. To solve the Navier-Stokes equation and the continuity equation, the fractional step method is adopted. On the other hand, the subdivision finite element method is used for solving the solid motion equation, with the capsule discretized into triangular elements with C^1-continuity by using the subdivision surface method. Details of the computational procedure can be found in Huang et al. [5].

For optical force calculation, we use the dynamic ray tracing method for a capsule with arbitrary deformation [6]. Optical force on the each node is obtained by

$$\boldsymbol{F}_{o,i} = \frac{n_m \boldsymbol{Q}_i}{c}\left(\boldsymbol{I}_i \cdot \boldsymbol{N}_{e,i}\right) \tag{3}$$

where n_{m} is the refractive index of fluid, and c is the speed of light in the free space, $\boldsymbol{I}$ is the Gaussian beam intensity, $\boldsymbol{N}_e$ is the unit normal direction on the surface, and $\boldsymbol{Q}$ is the dimensionless momentum transfer, which is calculated on the lower and upper surface by, respectively,

$$\boldsymbol{Q}_l = \boldsymbol{R}_i - R(\alpha)\boldsymbol{R}_r - n(1-R(\alpha))\boldsymbol{R}_t, \quad \boldsymbol{Q}_u = (1-R(\alpha))\left[\frac{n_i}{n_m}\boldsymbol{R}_i' - \frac{n_i}{n_m}R(\alpha')\boldsymbol{R}_r' - \frac{n_t}{n_m}(1-R(\alpha'))\boldsymbol{R}_t'\right], \tag{4}$$

where α (α') is the incident angle on the lower (upper) surface, $\boldsymbol{R}_i$ ($\boldsymbol{R}_i'$), $\boldsymbol{R}_r$ ($\boldsymbol{R}_r'$) and $\boldsymbol{R}_t$ ($\boldsymbol{R}_t'$) are respectively unit direction vector of the incident, reflected, and refracted ray on the lower (upper) surface, and n_i (n_t) is the refractive

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index of medium that the incident (refracted) ray passes. Incident angle and unit direction vectors are derived by using the ray-triangle intersection algorithm widely used on the computer graphics [6].

As a validation of the present method, we first simulate the motion of a rigid particle subjected to optical force in perpendicular direction with respect to fluid flow. Fig. 2 shows the particle trajectory that is well agreed with experiment [3]. The particle is moved following fluid flow and then is laterally migrated by the applied optical force. After escaping the illuminated region, the particle is again moved by fluid flow with its normal position retained. The maximum normal position indicates the lateral migration distance.

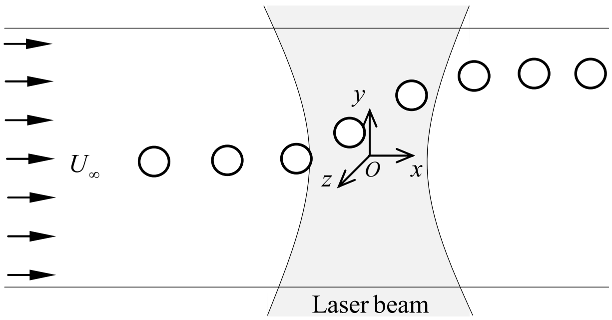

Fig.1. Schematic of the lateral migration of a capsule induced by the optical force in a uniform flow.

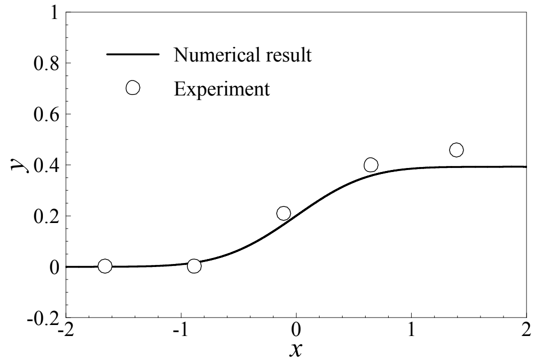

Fig. 2. Comparison of the particle trajectory between numerical simulation and experiment.

Next the elastic capsule is considered in our simulation. The optical force causes the deformation of the capsule and is reversely varied by the deformed shape. Since the deformation of a capsule is affected by elastic properties, we study the effects of elastic properties such as the surface Young's modulus (E_s) and the bending modulus (γ). To measure the deformation of the capsule, a deformation parameter is defined as $\varepsilon_{xy} = |L_1 - L_2|/(L_1+L_2)$, where L_1 and L_2 denote the two semi axis lengths of the deformed capsule in the center plane, i.e. z=0. Fig. 3 shows variations of ε_{xy} and the lateral migration distance with the surface Young's modulus and the bending modulus. It is found that the less deformed capsule is more migrated at the same condition. As shown in Fig. 3 (a) and (b), the lateral migration distance is significantly pronounced by increasing the surface Young's modulus but is only slightly increased by increasing the bending coefficient. However, the bending coefficient is also important for the migration of the capsule because it prevents buckling of the capsule which tends to occur at very low or high E_s [5]. The reason is that the tension force resists to the in-plane stretching while the bending force helps to maintain the surface curvature of the capsule. Hence, the migration distance of the capsule is mainly dependent on the surface Young's modulus but the capsule with bending rigidity is capable of being subjected to large optical forces without breakup.

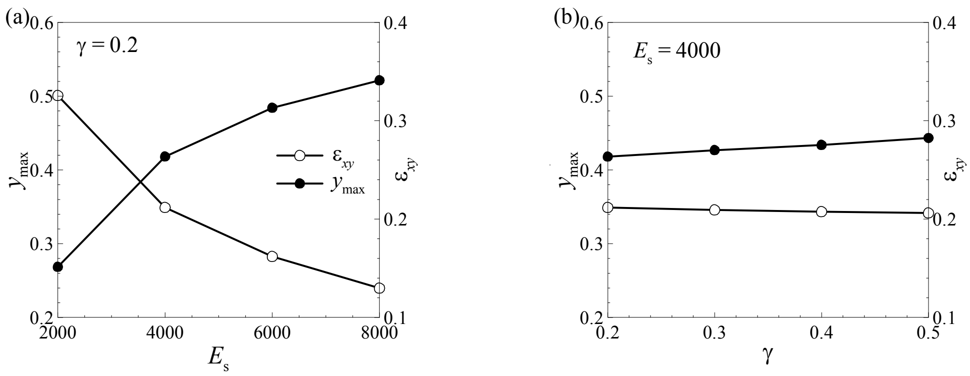

Fig. 3. Variations of the deformation parameter and the lateral migration distance with (a) the surface Young's modulus for γ=0.2 and (b) the bending modulus for E_s=4000.

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