AN ADAPTIVE IMMERSED BOUNDARY METHOD FOR FLUID STRUCTURE INTERACTION WITH BOUNDARY MASS

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Summary Extending the traditional immersed boundary method to cover the case of a massive boundary has required spreading the boundary mass out onto the fluid grid and then solving the Navier-Stokes equations with a variable mass density. A new way to give mass to the elastic boundary is proposed and shown that the new method can be applied to many problems for which the boundary mass is important. The new method does not spread mass to the fluid grid, and is easy to implement in the context of an existing IB method code for the massless case.

INTRODUCTION

The immersed boundary method has been widely applied to problems involving a moving elastic boundary that is immersed in fluid and interacting with it [1-2]. But most of the previous applications of the IB method have involved a massless elastic boundary and used efficient Fourier transform methods for the numerical solutions. The purpose of this paper is to propose a way to extend the immersed boundary (IB) method to a situation in which an elastic moving boundary has mass that cannot be neglected and to show an example of the application of this new methodology.

EQUATIONS OF MOTION

For a system comprised of a three-dimensional viscous incompressible fluid containing an immersed, elastic boundary with mass [4].

$$\rho_0\Big(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\Big)=-\nabla p+\mu\nabla^2\mathbf{u}+f, \tag{1}$$

$$\nabla\cdot\mathbf{u}=0, \tag{2}$$

$$\mathbf{f}(\mathbf{x},t)=\int \mathbf{F}(r,s,t)\delta(\mathbf{x}-\mathbf{X}(r,s,t))drds, \tag{3}$$

$$\frac{\partial \mathbf{X}}{\partial t}(r,s,t)=\mathbf{u}(\mathbf{X}(r,s,t),t)=\int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(r,s,t))d\,\mathbf{x}, \tag{4}$$

$$\mathbf{F}=\mathbf{F}_E+\mathbf{F}_K,\qquad \mathbf{F}_E=-\frac{\partial E}{\partial \mathbf{X}}, \tag{5}$$

$$\mathbf{F}_K(r,s,t)=K(\mathbf{Y}(r,s,t)-\mathbf{X}(r,s,t)), \tag{6}$$

$$M(r,s)\frac{\partial^2\mathbf{Y}}{\partial t^2}=-\mathbf{F}_K(r,s,t)-M(r,s)ge_3, \tag{7}$$

Where $\rho_0(\mathbf{x},t)$ is the uniform mass density of the fluid, μ the constant fluid viscosity. $\mathbf{u}(\mathbf{x},t)$ the fluid velocity, $p(\mathbf{x},t)$ the fluid pressure, $f(\mathbf{x},t)$ the force per unit volume applied by the immersed boundary to the fluid, $\mathbf{x}=(x,y,z)$ fixed Cartesian coordinates, t the time, δ a smoothed approximation to the Dirac delta function[4], K a large constant, F_K the restoring force, g the gravitational acceleration and e_3 a unit vector in the vertical direction. The unknown $\mathbf{X}(r,s,t)$ completely describes the motion of the immersed boundary. There are two boundaries immersed in a 3-D fluid: one is a massive boundary $\mathbf{Y}(r,s,t)$ having all the mass of the elastic immersed boundary, and the other is a massless boundary $\mathbf{X}(r,s,t)$. The massive boundary $\mathbf{Y}(r,s,t)$ has mass density $M(r,s)$, but does not interact with the surrounding fluid directly. $\mathbf{F}_E=\mathbf{F}_E(r,s,t)$ the force density applied by the immersed boundary to the fluid, and the elastic contribution to this force density is given by the variational derivative $-\partial E/\partial\mathbf{X}$ of the elastic energy functional $E\left[\mathbf{X}(\quad,\quad,t)\right]$. In a 2-D space, we consider an immersed boundary as a 1-D rod and use two elasticities: one that resists stretching and compression, and another that resists bending:

$$E\left[\mathbf{X}(\cdot)\right]=\frac{1}{2}c_s\int\Big(\left|\frac{\partial\mathbf{X}}{\partial s}\right|-1\Big)^2 ds+\frac{1}{2}c_b\int\left|\frac{\partial^2\mathbf{X}}{\partial s^2}\right|^2 ds \tag{8}$$

where c_s and c_b are constants. The numerical method used is described in detail in Ref.[4].

NUMERICAL EXAMPLE

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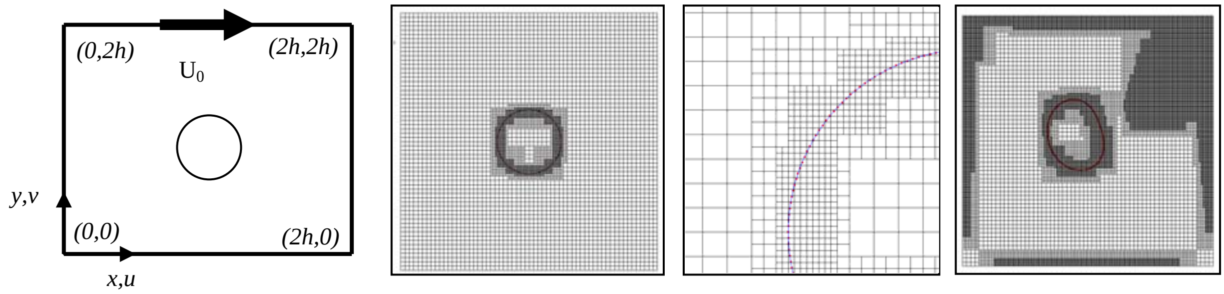
A ball immersed in a lid-driven cavity is shown in Fig. 1. Steady motion of the top wall induces flow, while the other walls remain stationary. The velocity of the top wall is assumed to be of the form: $u(x,h)=U_0(1-(x/h-1)^{18})^2$. where U_0 is the maximum velocity, $2h$ is the length of side. The radius of ball is 0.25h, Results for $\mathbf{Re}=\rho_0 U_0(2h)/\mu$ =12000 are presented in what follows. c_s =5000, c_b =19.

The strong swirl flow induced the ball movement and deformation against time is illustrated in Fig. 2.



(a) Geometry （b）Initial mesh (c) Local mesh (d) Adaptive mesh

Fig. 1. The geometry configure and computational mesh

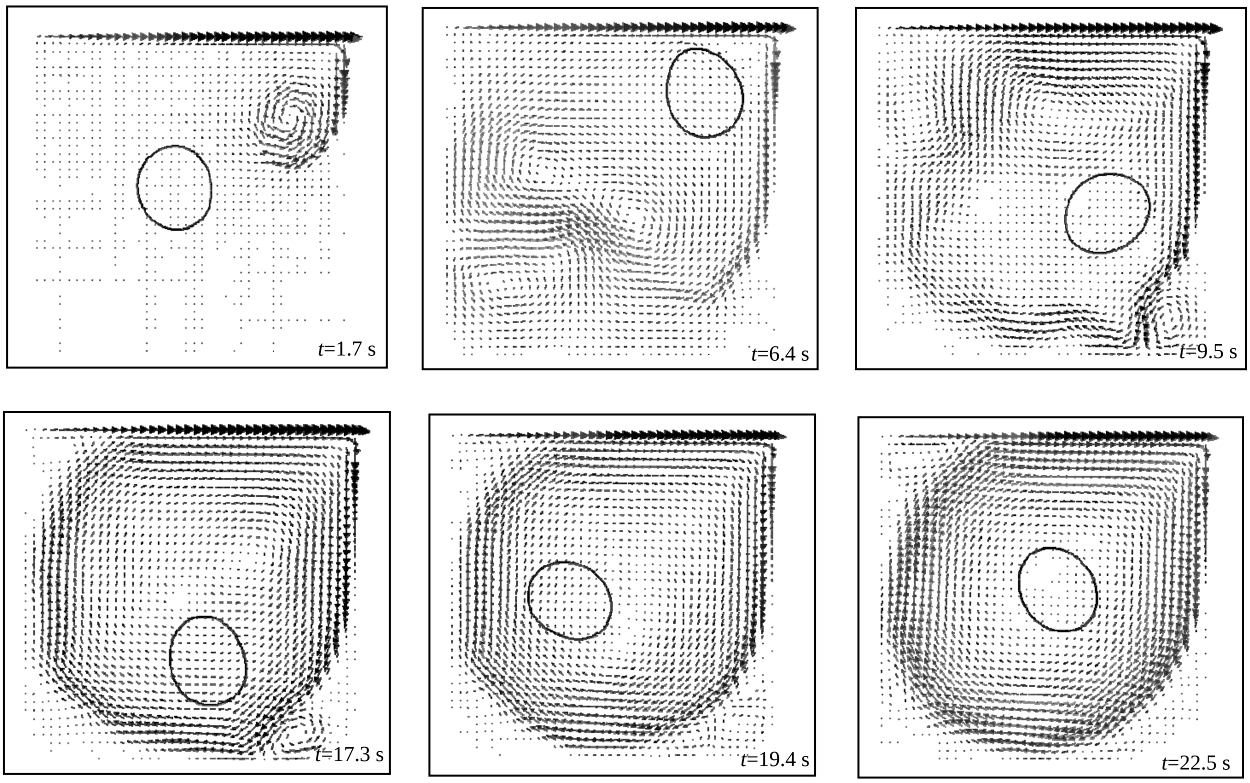


Fig. 2. The velocity vector and position of ball against time

CONCLUSIONS

Extending the traditional immersed boundary method to cover the case of a massive boundary for which the boundary mass is important, a numerical test is implemented to verify the new immersed boundary method, and the result show the validity of the method.

References

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