SIMULATION OF THE INTERACTION BETWEEN COMPRESSIBLE FLUID AND THIN ELASTIC PLATE BY THE MODIFIED GHOST FLUID METHOD

Liang Xu*,**a), Tiegang Liu* & Zhaoyong Ni**
*LMIB and School of Mathematics and Systems Science, Beijing University of Aeronautics and Astronautics, Beijing, 100191, China
**China Academy of Aerospace Aerodynamics, Beijing, 100074, China

Summary This work is devoted to extending the modified ghost fluid method (MGFM) to treat compressible fluid coupled with thin elastic plate and applying it to the underwater explosions (UNDEX) problem. By solving shock relationship and thin-elastic-plate equation together to predict the ghost fluid states and the plate load, this approach not only ensures numerical stability and maintains the advantages of simplicity and high efficiency, but also provides a more accurate interface boundary condition. Evolution of strong shock impacting on thin elastic plate and dynamic response of the plate are investigated. Results disclose that the MGFM provides a convenient and effective way for handling the coupling between flow and plate.

INTRODUCTION

Research on fluid-structure interaction (FSI) is important and attractive in a number of applications. Generally speaking, the loosely-coupled method is relatively simple and convenient in numerical simulation. Currently popular high-order fluid and solid solvers can be easily coupled together to simulate FSI problems. However, the nonlinear interaction at the interface is hard to be taken into consideration faithfully, and thus numerical instability is induced, especially when the structure is under a strong shock wave impact.

Recently, the modified ghost fluid method (MGFM) [1], originally applied to treat multi-fluid coupling, has been employed to model FSI. The interface boundary conditions are accurately imposed at the same moment via constructing and solving a multi-medium Riemann problem. Based on this treatment, the solid is modeled as fluid-like material under strong impact and governed by the Euler equations [2]. Generally, it is more common to describe the solid in a Lagrangian framework. For example, the Navier equations based on the theory of elasticity for small deformation is employed to model the elastic solid [3]. However, this treatment is not suitable for thin plate structure. In this work, the MGFM is further extended to compute the fluid-plate interaction problem. Our interest is primarily in simulating the process of strong shock wave generated by UNDEX impacting on a thin plate.

GOVERNING EQUATIONS AND FLUID-PLATE INTERFACE TREATMENTS

The governing equations for 2D inviscid and compressible flow are Euler equations, which can be written as

$$\frac{\partial U}{\partial t}+\frac{\partial F(U)}{\partial x}+\frac{\partial G(U)}{\partial y}=0, \tag{1}$$

where U, $F(U)$ and $G(U)$ are the vectors of conserved variables and fluxes, given respectively by $U=[\rho,\rho u,\rho v,E]^T$, $F(U)=[\rho u,\rho u^2+p,\rho uv,(E+p)u]^T$, $G(U)=[\rho v,\rho uv,\rho v^2+p,(E+p)v]^T$. Here ρ is the density, u and v are the velocity components in the respective $x-$ and $y-$ directions, p is the pressure, and E is the total energy per unit volume. The EOS for closure of system (1) can be expressed as $p=(\gamma-1)\rho e-\gamma B$. Here γ and B are set to 2.0 and 0.0 for explosive gas, to 7.15 and 3.309×10^8 Pa for water accordingly. For the solid model, the governing differential equation for the transverse displacement w is given by the thin-elastic-plate (or Kirchhoff) equation

$$D\nabla^2\nabla^2 w+\rho_s h\frac{\partial^2 w}{\partial t^2}=q, \tag{2}$$

where D is the flexural rigidity of the plate defined by $D=(Eh^3)/(12(1-\nu^2))$. In the thin-elastic-plate equation, E, ν, h and ρ_s are constants, which respectively denote the elastic modulus (Young's modulus), Poisson's ratio, thickness and density of the plate. q is the loading function. ∇^2 is the 2D Laplacian operator.

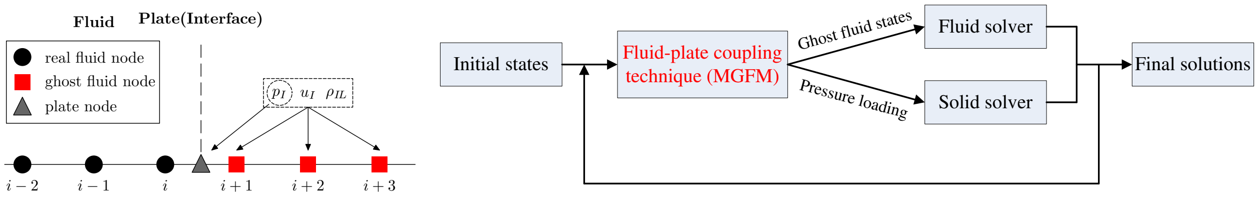
We assume that the fluid is located on the left side of the interface (thin plate). In the MGFM, We need to predict the interface state by coupling a nonlinear characteristic from the fluid and the thin-elastic-plate equation. This can be done by solving the shock relationship and the discrete form of (2) together. That is to say, we solve the coupled system

$$\begin{cases}\dfrac{p_I-p_l}{W_l}+(u_I-u_l)=0, \quad W_l=\rho_l c_l\sqrt{1+\dfrac{\gamma_l+1}{2\gamma_l}\dfrac{p_I-p_l}{p_l+B_l}},\\ \dfrac{p_I-(p_0+D\nabla_h^2\nabla_h^2 w)}{\rho_s h/\Delta t}-(u_I-u_I^n)=0,\end{cases} \tag{3}$$

to predict the fluid-plate interface state. Here, $\nabla_h^2\nabla_h^2 w$ denotes one discrete form in space. p_0 can be taken as the atmospheric pressure or zero, as required.

a) Corresponding author. E-mail: xuliang@smss.buaa.edu.cn.

After the predicted pressure and velocity at the interface are solved, the predicted density can be obtained by utilizing the shock relationship. Then, as depicted in Fig. 1(a), we could define the ghost fluid states and the plate load: for the fluid, the predicted pressure p_I, velocity u_I and density ρ_{IL} are defined as the ghost fluid states; for the plate, only the result of $p_I - p_0$ is defined as the load. Usually, a narrow band of 3 to 5 grid points as ghost cells is defined in the vicinity of the plate. Such treatment is also classified as a partitioned algorithm, but overcomes the shortcomings of loosely-coupled method via predicting the interface state. In addition, it is also suitable for a fixed Eulerian mesh construction, avoiding any mesh deformation or remeshing step. This implementation is depicted in Fig. 1(b).



(a) 1D schematic diagram for the MGFM.

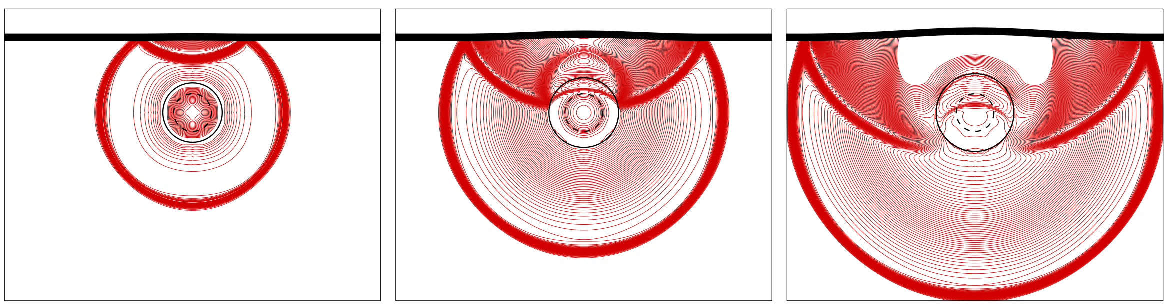
(b) Diagram of coupling between fluid and solid solvers.

Figure 1. llustration of the MGFM treatment for the fluid-plate interaction.

NUMERICAL EXPERIMENTS AND DISCUSSIONS

In the following calculations, we assume a thin plate is initially located at the straight line $y = 2$m with thickness of 0.25m. This plate is set to be a stainless steel plate and its material properties are $E = 2.05 \times 10^{11}$Pa, $\nu = 0.3$, $\rho_s = 7800\text{kg/m}^3$. The initial real computational domain for fluid is a rectangular region with $\text{X} \times \text{Y} \in [-5\text{m}, 5\text{m}] \times [-5\text{m}, 2\text{m}]$. An explosive gas bubble with radius of 0.5m is located at the origin $(0.0, 0.0)$ in the water. The initial flow parameters for gas bubble are $\rho_g = 1270.0\text{kg/m}^3$, $p_g = 8.29 \times 10^8$Pa. The initial flow parameters for water are $\rho_w = 1000.0\text{kg/m}^3$, $p_w = 1.0 \times 10^5$Pa. Both the bubble and water are static initially. A total of 321×225 uniform grid points are distributed in the real fluid computational domain, and 21×21 uniform grid points are distributed in the plate computational domain. The strategy in [1] is adopted as the treatment for gas bubble-water coupling. The nonreflective boundary conditions are used for all the fluid outside boundaries. To capture the moving gas-water interface, the level set technique is utilized.

In order to clearly demonstrate the evolution of strong shock impacting on thin plate and the dynamic response of the thin plate, the pressure contours at different stages of earlier times are shown in Fig. 2. The dashed circle represents the initial bubble while the solid circle represents the expanded bubble. Fig. 2(a) shows that a strong underwater shock impacts the plate, resulting in the incident shock reflected from the fluid-plate interface. Fig. 2(b) shows that the reflected wave, as a result of contacting the expanded bubble, is divided into two parts: one is transmitted into the air bubble, while the other is reflected as a rarefaction wave propagating along the opposite direction. After the reflected rarefaction wave reaches the bottom surface of the plate, the wave-plate interaction causes a low pressure region to be formed. As time goes on, the pressure there keeps on decreasing, leading to the incipience of cavitation. See Fig. 2(c). The thin plate, under the lasting impact of load, has presented a certain degree of bending. Because the material used is steel with larger elastic modulus, the deformation rate is relatively small.



(a) 1.0 ms.

(b) 1.7 ms.

(c) 2.4 ms.

Figure 2. Pressure contours at different times.

References

[1] T. G. Liu, B. C. Khoo, K. S. Yeo, Ghost fluid method for strong shock impacting on material interface, J. Comput. Phys., 190 (2003), 651-681.

[2] T. G. Liu, W. F. Xie, B. C. Khoo, The modified ghost fluid method for coupling of fluid and structure constituted with hydro-elasto-plastic equation of state, SIAM J. Sci. Comput., 30 (2008), 1105-1130.

[3] T. G. Liu, A. W. Chowdhury, B. C. Khoo, The modified ghost fluid method applied to fluid-elastic structure interaction, Adv. Appl. Math. Mech., 3 (2011), 611-632.