OBLIQUE WAVES LIFT THE FLAPPING FLAG

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Summary The flapping of the flag is a classical model problem for the understanding of fluid/structure interaction : how does the flat state lose stability? and why do the nonlinear effects induce hysteretic behavior? We show in this letter that in contrast to the commonly studied model, the full three-dimensional flag with gravity has no stationary state whose stability can be formally studied: the waves are oblique and must immediately be of large amplitude. The remarkable structure of these waves result from the interplay of weight, geometry and aerodynamic forces. This pattern is a key element in the force balance which allows the flag to hold and fly in the wind: large amplitude oblique waves are responsible for lift.

INTRODUCTION

When asked to draw a flapping flag, most—scientists and laymen alike—will produce something in the spirit of figure 1a: propagating waves with vertical crests. Figure 1f is a typical flag as you could observe in a windy day: the waves are oblique.

In 1879, Lord Rayleigh touches upon the flapping of flags and sails in his influential paper on unstable fluid motion [Rayleigh(1879)]. The analogy between flag and shear layer stability is immediate since Helmholtz and Kelvin have developed a framework for stability analysis based on surfaces of discontinuity now known as vortex sheets [Helmholtz(1868), Thomson(1871)]. The fluid motion of a shear layer consists of two semi-infinite bodies of constant velocities separated by such an immaterial mobile surface. Quantifying the motion of this surface under the dynamic pressure forces is instrumental in the description of the Kelvin–Helmholtz instability. When this moving surface is bestowed with the additional physical properties of mass, tension and rigidity, we have the flag. This conception of a flag has inherited the simplifications which are relevant for the description of the Kelvin–Helmholtz instability: it is infinitely long to allow for description of its motion into harmonic waves, and it is two-dimensional with waves propagating with the stream.

The stability analysis for waves short with respect to the flag length has now reached a standard [Païdoussis(1998)]: applied tension, be it external or induced from fluid drag, is stabilizing, as is bending rigidity, whereas instability increases for longer wave lengths and heavier cloth. The simulations in [Connell and Yue(2007)] confirm the relevance of this local stability analysis for two-dimensional soft flags. Large amplitude analysis shows in addition an hysteretic behavior which is now understood as typical for these systems. This was shown for instance in [Zhang et al.(2000)] by observation of the flapping of a silk thread in a falling soap film.

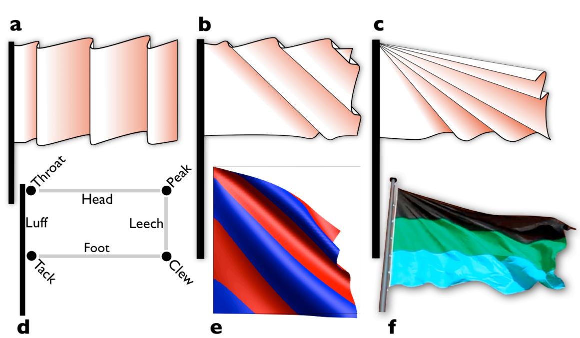

Figure 1. General representation of the flag which is discussed in the paper. a) popular conception of the flapping flag with vertical wave crests. b) A first pattern of oblique waves. c) An alternative pattern of accordion waves. d) Naming of the flag's part in analogy to quadrangular sails. e) Numerical simulation of a flag in the wind by [Huang and Sung(2010)]. f) A flapping flag in a windy day.

When on the other hand considering more rigid media, like a cardboard flag for which the rigidity is such that only waves of length comparable to that of the flag can be unstable [Argentina and Mahadevan(2005), Eloy et al.(2008)] we see that the dynamics of the vorticity in the free wake will be important for the stability: the flapping induces shedding of vortices, and these vortices act in turn through the pressure on the surfaces of the flag such as to organize a synchronized flapping behavior reminiscent of typical fluid/structure instabilities like aeolian tones [Rayleigh(1915), Jeans(1968)], or airplane wing flutter [Theodorsen(1935)]. For a review of flapping and bending bodies in fluid flow, see [Shelley and Zhang(2011)].

These studies are instrumental steps in conceiving a fundamental model system—as simple as possible—to form the basis of understanding of fluid/flexible structure interaction. We may now take the occasion to turn back to the original inspiration of the flag, and see whether the picture is complete or if the simplifications that were made necessary by the

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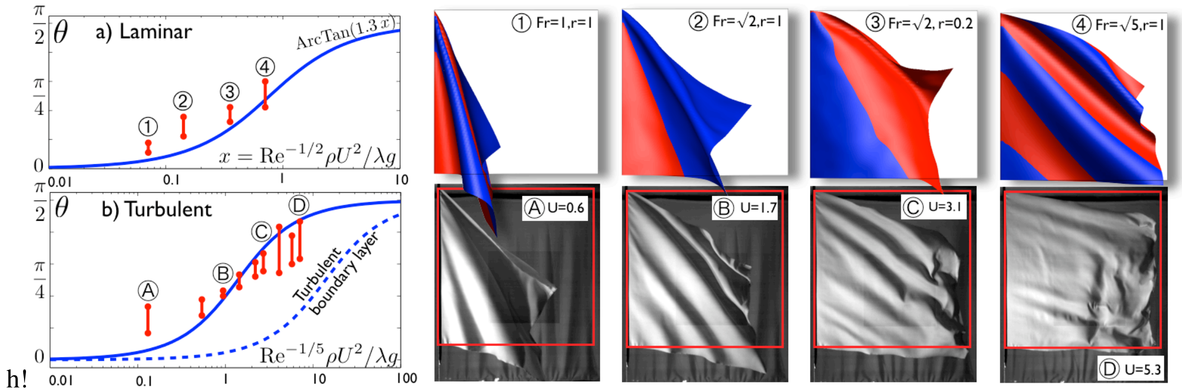

Figure 2. Measurement of the wave angle. a) Low Reynolds number; The wave angle obtained by a balance of aerodynamic drag and weight for a laminar flow, compared to numerical simulations from [Huang and Sung(2010)]. The flag is colored in blue/red for positive/negative deflections of the cloth. b) High Reynolds number; wave angle estimated from turbulent boundary layer drag (dashed line) and with a force magnified to account for extra drag due to large amplitude wave motion (solid line). The flag motion is shown in a movie included as a supplementary material.

search of a quantitative modeling have overlooked some characteristic phenomena. In this, the most striking observation is made by [Huang and Sung(2010)] in a numerical simulation, showing that the same flag in the same wind remains flat if weightless, but flaps with oblique waves otherwise. For another observation of oblique waves in numerical simulations, see [Kim and Peskin(2007)].

OBLIQUE WAVES

Fluid drag generates a tension mitigating the instability, but it is also the only force which can oppose collapse due to weight. The complete surface of the cloth is subject to an oblique force composed of the downward weight and the backward drag. It is clear that horizontal drag can only partly oppose the weight, but it is not easy to determine which is the exact zone of the flag that will be able to hold flat. On the other hand we may imagine an analog configuration for which the answer is straightforward. This is the case for a flag hanging under its weight only, but from a tilted pole. The region of the cloth which in this configuration is not hanging flat below the supporting pole will fall down and wrinkle. Putting back the flag in its original vertical pole configuration, we see that the frontier separating the wrinkling zone and the flat zone is now oblique. The angle θ of this obliquity is obtained from the vector sum of the horizontal viscous drag and the vertical weight

$$\tan(\theta) = f/\lambda g$$

if λ is the surface weight of the cloth, g the acceleration of gravity, and f is the intensity of the viscous pull.
We have weasured the angle of the waves in numerical simulations as well as in a wind tunnel experiment, see figure 2. The observed angles are compared with the angles obtained from the laminar viscous pull with a law in $Re^{-1/2}$, and for the turbulent case with the law in $Re^{-1/5}$, see [Schlichting and Gersten(1999)]. For a more detailed analysis of the phenomenon, please see [Hoepffner and Naka(2011)].

References

[Huang and Sung(2010)] W.-X. Huang and H. J. Sung, J. Fluid Mech. **653**, 301 (2010).
[Rayleigh(1879)] Rayleigh, Proc. Lond. Math. Soc. **10**, 4 (1879).
[Helmholtz(1868)] H. Helmholtz, Phil. Mag. series 4 **36**, 337 (1868).
[Thomson(1871)] W. Thomson, Phil. Mag. series 4 **42**, 362 (1871).
[Païdoussis(1998)] M. P. Païdoussis, *Fluid-Structure Interactions: Slender Structures and Axial Flow.* (Academic Press, 1998).
[Connell and Yue(2007)] B. S. H. Connell and D. K. P. Yue, J. Fluid Mech. **581**, 33 (2007).
[Zhang et al.(2000)] J. Zhang, S. Childress, A. Libchaber, and M. Shelley, Nature **408** (2000).
[Argentina and Mahadevan(2005)] M. Argentina and L. Mahadevan, PNAS **102**, 1829 (2005).
[Eloy et al.(2008)] C. Eloy, R. Lagrange, C. Souilliez, and L. Schouveiler, J. Fluid Mech. **611**, 97 (2008).
[Rayleigh(1915)] Rayleigh, Phil. Mag. **29**, 433 (1915).
[Jeans(1968)] J. Jeans, *Science & Music* (Dover publications, inc., 1968).
[Theodorsen(1935)] T. Theodorsen, Tech. Rep. 496, NACA (1935).
[Shelley and Zhang(2011)] M. J. Shelley and J. Zhang, Annu. Rev. Fluid Mech. **43**, 449 (2011).
[Kim and Peskin(2007)] Y. Kim and C. S. Peskin, Phys. Fluids (2007).
[Moretti(2003)] P. M. Moretti, Int. J. Acoustics and Vib. **8**, 227 (2003).
[Seidman(2011)] D. Seidman, *The Complete Sailor: Learning the Art of Sailing* (International Marine Publishing Co, 2011).
[Schlichting and Gersten(1999)] H. T. Schlichting and K. Gersten, *Boundary-Layer Theory* (Springer-Verlag Berlin, 1999).
[Hoepffner and Naka(2011)] T. Hoepffner and A. Naka, Physical Review Letters **107**, 194502 (2011).