ORBITAL MOTION OF A LONG FLEXIBLE CYLINDER SUBJECT TO VORTEX-INDUCED VIBRATIONS

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Summary On the basis of detailed numerical simulation results, we investigate the synchronization of the in-line and cross-flow vortex-induced vibrations of a long flexible cylinder placed in sheared current, in the cases of narrowband and broadband structural responses. We analyze the occurrence of preferential trajectories in the regions of wake-body synchronization and the phenomenon of drifting phase difference between the in-line and cross-flow vibrations.

VORTEX-INDUCED VIBRATIONS OF LONG FLEXIBLE BODIES

Vortex shedding in the wake of a bluff body immersed in a cross-flow induces unsteady forces on the body, and can lead to vibrations if the body is flexible or flexibly mounted. The phenomenon of vortex-induced vibrations (VIV), occurs in a number of physical problems, especially in the context of long flexible cylindrical structures employed in ocean engineering, such as risers or marine cables. The case of a rigid circular cylinder free to move, or forced to oscillate in the cross-flow (CF) direction within a uniform current, has served as a canonical problem to investigate VIV mechanisms [1, 2, 3]. It has been shown that self-excited, self-limited, large amplitude oscillations occur when the vortex shedding and the structural vibration frequencies coincide, a condition of wake-body synchronization referred to as *lock-in*. Significant differences in the CF response and in the fluid forces have been reported when the rigid cylinder is also allowed to move in the in-line (IL) direction [4, 5].

The increased complexity of the VIV phenomenon in the case of slender flexible bodies immersed in non-uniform currents has been highlighted in recent experimental and numerical works [6, 7]. Mono-frequency as well as multi-frequency structural responses may occur in this context. The shape and orientation of the cylinder trajectory in the plane perpendicular to its axis are determined by the phase difference between the IL and CF vibrations, which can vary along the span for a slender deformable body.

The objective of this work is to analyze the principal features of the synchronization between the IL and CF narrowband VIV of a long flexible cylinder placed in shear flow, with an emphasis on the link between body trajectory and fluid-structure interaction mechanisms, and to quantify the validity of this analysis in the case of broadband VIV, where both high and low structural wavenumbers are excited.

We consider a flexible cylinder of circular cross-section and length, L, to diameter, D, aspect ratio 200, pinned at both ends, free to oscillate in the in-line and cross-flow directions and immersed in a sheared current, as illustrated in figure 1. Two types of sheared profiles, linear and exponential, are used to trigger narrowband and broadband responses. In both cases, the ratio between the maximum and minimum oncoming flow velocity is set to 3.67 and the maximum Reynolds number, Re_m, is in the range $110 - 1100$. The flow is predicted by direct numerical simulation of the three-dimensional Navier-Stokes equations. The structural dynamics is governed by a tensioned beam model. The structure to displaced fluid mass ratio, m, is in the range $3 - 6$. Tension and bending stiffness are selected so that high wavenumber vibrations develop. The coupled fluid-structure problem is solved by the Nektar code that is based on spectral/hp element method [8] and that has been validated in previous works concerning similar configurations [7].

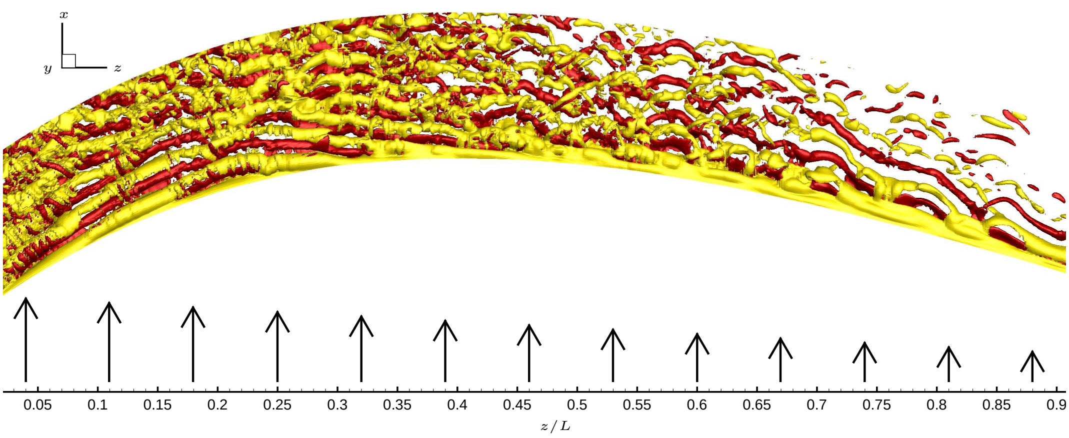

Figure 1. Instantaneous iso-surfaces of the spanwise vorticity, $\omega_z = \pm 0.3$, at $\mathrm{Re}_m = 330$ and $m = 3$; part of the computational domain is plotted: $[0.6D, 25D]$ downstream of the cylinder. Arrows indicate the sheared oncoming flow.

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ORBITAL MOTION, LOCK-IN CONDITION AND FLUID-STRUCTURE ENERGY TRANSFER

The typical structural vibrations observed in the IL and CF directions are combinations of standing and traveling wave patterns. In both cases of mono- and multi-frequency responses, a ratio of 2 can be identified between the IL and CF excited frequencies. In the case of narrowband responses, the frequency ratio of 2 leads to 'figure eight' trajectories in the (x, y) plane as illustrated in figure 2(a). The phase difference between the IL and CF vibrations is defined as $\Phi_{xy} = [p\phi_x - q\phi_y, \mod 360^o]$, where ϕ_x and ϕ_y are the instantaneous IL and CF response phases and $(p, q) = (1, 2)$ since displacement synchronization is studied for a frequency ratio of 2. Values of Φ_{xy} in the range $0^o - 180^o$ ($180^o - 360^o$ respectively) correspond to 'figure eight' orbits where the beam moves upstream (downstream respectively) when reaching the cross-flow oscillation maxima. These two types of trajectories are referred to as 'counter-clockwise' (CC) and 'clockwise' (C) respectively [5], as indicated in figure 2(a).

Typical histograms of Φ_{xy} issued from narrowband response time series are plotted along the span in figure 2(b). Due to the pronounced standing wave character of the vibrations near $z = 0$, transitions can be noted between CC and C trajectories near the nodes of the in-line response. In the region where purer traveling waves develop, the phase difference exhibits a well-defined, nearly linear drift towards higher angles as z increases.

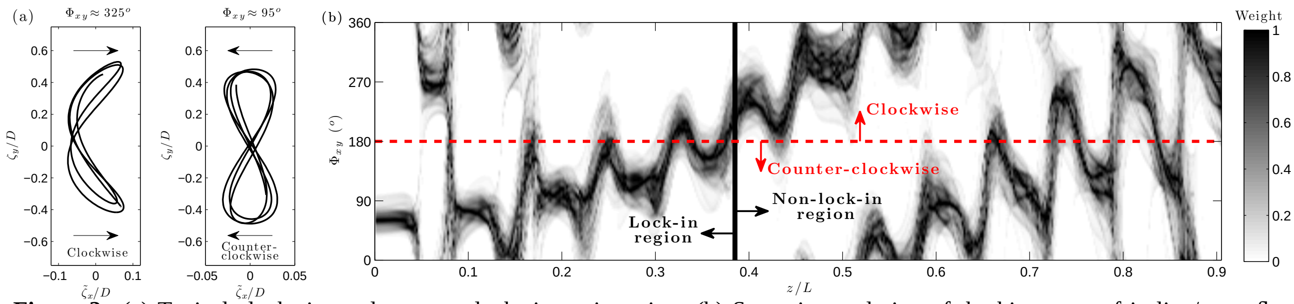

Figure 2. (a) Typical clockwise and counter-clockwise trajectories. (b) Spanwise evolution of the histogram of in-line/cross-flow motion phase difference, at $\text{Re}_m = 330$ and $m = 3$. In (b), the limit of the lock-in region is indicated by a vertical black line and the transition from counter-clockwise to clockwise trajectories ($\Phi_{xy} = 180^o$) by an horizontal red dashed line.

For long flexible structures, the lock-in condition can be defined at each spanwise location by the synchronization of the local vortex shedding frequency with the local CF vibration frequency; otherwise, the condition is referred to as non-lock-in. In the case of narrowband responses, the lock-in region is located on the high velocity side, as indicated in figure 2(b). It can be noted that CC orbits dominate the synchronization pattern within the lock-in region. Energy is transferred from the fluid to the body under the lock-in condition; as a consequence, the flow principally excites the structural vibrations through CC orbits while C trajectories are associated with damping fluid forces.

In broadband response case where both high- and low-wavenumber vibrations are excited along the span, the regions of energy transfer from the flow to the structure also coincide with the occurrence of the lock-in condition. However, lock-in exhibits a contrasted behavior in this case: the synchronization condition is *distributed* along the length of the body, in spite of the sheared inflow, and is not limited to the high velocity zone. By considering each pair of IL/CF excited frequencies occurring with a ratio of 2 separately, we show that the corresponding phase difference is mainly associated with CC orbits in the lock-in region, which generalizes the previous analysis to the case of broadband VIV.

The spanwise drift of the IL/CF motion phase difference, identified in the case of narrowband responses, occurs within and outside the lock-in region (figure 2(b)) and leads to a regular alteration of the body trajectory along its length. By means of a simplified model composed of two waves traveling in opposite directions, in the IL and CF directions, we show that the phase drift occurs when a ratio different from 2 is established between the IL and CF excited wavenumbers. The departure from a wavenumber ratio of 2 is partly due to the nature of the structure that is characterized by a non-linear dispersion relation but we also highlight the influence of the effective added mass induced by the fluid forces in phase with the body acceleration.

Such drift of the IL/CF phase difference is not observed when the structure exhibits broadband responses. In this case, the excited frequencies combine and lead to a nearly constant phase difference along the length of the body.

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