NUMERICAL INVESTIGATION OF INTERACTIONS BETWEEN TWO PARTICLES WITH DIFFERENT SIZES

Liang Wang*, Zhaoli Guo*[a)], Jianchun Mi**& Chuguang Zheng*
*National Laboratory of Coal Combustion, Huazhong University of Science and Technology, Hubei, Wuhan 430074, China
**State Key Laboratory for Turbulence and Complex Systems, College of Engineering, Peking University, Beijing 100871, China

Summary The interactions between two circular particles sedimenting in a two-dimensional infinite channel is numerically investigated by using the lattice Boltzmann equation with a multiple-relaxation-time collision model. The main emphasis in this work is to investigate the effect of the longitudinal distance and diameter ratio between two particles on the flow pattern during sedimentation. With two cases of initial positions, the results reveal the transitions between the drafting, kissing and tumbling (DKT) phenomenon.

INTRODUCTION

Sedimentation of particles in a viscous fluid has attracted much attention in theoretical and experimental study for many years, and a variety of numerical methods have been developed and extensively used to study particle-fluid systems. Fundamental mechanisms of fluid-particle and particle-particle interactions are very important for accurately predicting such flows behaviour. The sedimentation of two circular particles serves as the most simple problem to study these two types of interactions. To this point, many experimental and numerical studies have been carried out to investigate the behaviour of sedimentation of circular particles [1, 2, 3, 4, 5]. For the interactions between two sedimenting particles, many intrinsic factors including particle density, shape and surface properties play a role in particle settling velocity, segregation or migration. But in most cases, the particle size plays the dominant role in the real engineering applications. So far, the majority of numerical studies of interactions between two sedimenting particles concentrate on uniform particle sizes. The attention paid to the effect of diameter ratio on particle motion and interactions has been very limited. For two lighter-than-fluid particles in a finite fluid-filled rotating cylinder, Mukundakrishnan *etal.* considered the effect of densities and sizes of particle pairs. Shao *etal.* simulated the sedimentation of two circular particles on different size situations using the DLM/Fictitious Domain method [11]. However, it is noticed that the analysis of the effect of diameter ratio considered in Ref. [11] restrict to a fixed two-particle configuration in the channel. This study may be not enough to fully reveal the mechanism of the interactions of two sedimenting particles with different sizes. A more detailed study on the effect of diameter ratio would be required. To our knowledge, few attention have been paid to address this issue. Therefore, the main focus of our present effort is to investigate the effects of different sizes on the sedimentation of two circular particles.

As an alternative computational approach for Navier-Stokes (NS) equations, the lattice-Boltzmann method (LBM) [6, 7, 8, 9, 10] has also been shown to be an effective tool for simulating particulate suspensions. In this paper, we will use the LBM to simulate the sedimentation of two circular particles in a two-dimensional infinite channel. It has been reported that the multiple-relaxation-time (MRT) model has apparent advantages, such as better numerical instability for low viscosity, over the Bhatnagar-Gross-Krook (BGK) model. Therefore, the numerical simulations of this paper are limited to the MRT model.

NUMERICAL METHOD

In LBE for particulate flows, the evolution of MRT-LBE can be expressed as [12, 13]

$$f_i(\boldsymbol{x}+\boldsymbol{c}_i\delta_t, t+\delta_t) - f_i(\boldsymbol{x},t) = -\sum_j \big(\boldsymbol{M}^{-1}\boldsymbol{S}\boldsymbol{M}\big)_{ij}\big[f_j(\boldsymbol{x},t) - f_j^{(eq)}(\boldsymbol{x},t)\big], \tag{1}$$

where $f_i(\boldsymbol{x},t)$ is the fluid distribution function for the particle moving with discrete velocity $\boldsymbol{c}_i$ at time t and position $\boldsymbol{x}$, $f_j^{(eq)}(\boldsymbol{x},t)$ is the local equilibrium distribution function (EDF), and δ_t is the time step. For the D2Q9 model, $\boldsymbol{M}$ is a 9×9 matrix which linearly transforms the distribution functions to their moments: $\boldsymbol{m} = \boldsymbol{M}\cdot\boldsymbol{f}, \quad \boldsymbol{f} = \boldsymbol{M}^{-1}\cdot\boldsymbol{m}$, where $\boldsymbol{f} = (f_0, f_1, \ldots, f_8)^T$ and T denote a column vector and the transposition operator, respectively. $\boldsymbol{S}$ is the corresponding collision matrix in moment space, which is a diagonal matrix of relaxation rates $\{s_i\}$: $\boldsymbol{S} = diag(s_0, s_1, \ldots, s_8)$. The density ρ and velocity $\boldsymbol{u}$ of fluid are defined by $\rho = \sum_i f_i$ and $\rho\boldsymbol{u} = \sum_i \boldsymbol{c}_i f_i$. In the D2Q9 model, the details of the above MRT model can be found in Refs. [12, 13].

The boundary condition on the surface of a solid particle, i.e., the no-slip condition, is realized by the bounce-back rule proposed by [6] and [14]:

$$f_{i'}(\boldsymbol{x}, t+\delta_t) = f_i^*(\boldsymbol{x},t) - 2\omega_i\rho\frac{\boldsymbol{c}_i\cdot\boldsymbol{u}_b}{c_s^2}, \tag{2}$$

[a)] Corresponding author. E-mail: wlsa0612@gmail.com, zlguo@hust.edu.cn

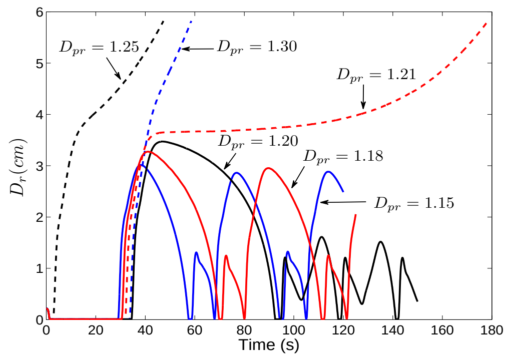

Figure 1. Effect of variations of the diameter ratio in the range $1-2$ on the DKT process.

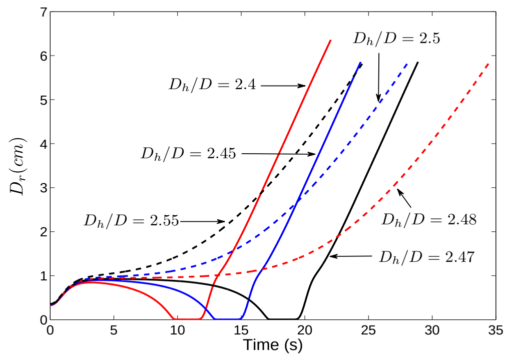

Figure 2. Effect of initial longitudinal distance D_h on the interaction of two particles at $D_{pr} = 2.0$.

where $\boldsymbol{x}$ is the position of the node adjacent to the solid surface with velocity $\boldsymbol{u}_b$, i' denotes the reflection direction and i the incident direction. In the present work, we will adopt Ladd's method [6, 14] for the numerical simulations.
For fluid-particle interactions, the momentum-exchange method is used to calculate the hydrodynamic force exerted on a solid particle, and the translational and the angular motion of the particle are determined by Newton's law. For particle-particle and particle-wall interactions, we will adopt the collision model proposed in Ref. [15].

MAIN RESULTS

We have validated our LB code by comparison with the existing results for the sedimentation of two identical particles in an infinite channel. In what follows, the main results of two non-identical particles in size will be presented at two cases of initial positions, i.e., the larger particle is located above the smaller particle, and the smaller particle above the larger one. As pointed out in Introduction, the effect of initial distance D_h and diameter ratio D_{pr} between two particles on their interactions will be investigated. Three kinds of distances between two particles, i.e., the difference of transverse coordinates D_x and longitudinal coordinates D_y of the two particle centers, and the gap between the two surfaces D_r, are introduced to analyze their interactions, especially the DKT process.

Case 1. The larger particle above the smaller one

It is clearly known that the larger particle sediments faster than the smaller one. Thus, one can infer that whatever the initial longitudinal distance between two particles with different sizes are set to be, the larger particle is bound to catch up with the smaller one originally in the lower position as time proceeds, and the DKT phenomenon will take place subsequently. Further, we find that the two particles undergo the DKT process only once.

Effect of diameter ratio

As can be seen from figure 1, for all the cases of diameter ratios the curves of the gap almost contact with the line $D_r = 0$ once, which undoubtedly implies the occurrence of the DKT process. More important, it is observed that the two particles undergo repeated DKT process when the diameter ratio $D_{pr} \leq 1.2$, while for $D_{pr} > 1.2$ the two particles separate from each other, and the gap between them increases with time. In addition, it is also found that the repeated DKT process occurs more frequently with decreasing D_{pr}, as has been shown by [11].

Case 2. The smaller particle above the larger one

Effect of the initial longitudinal distance

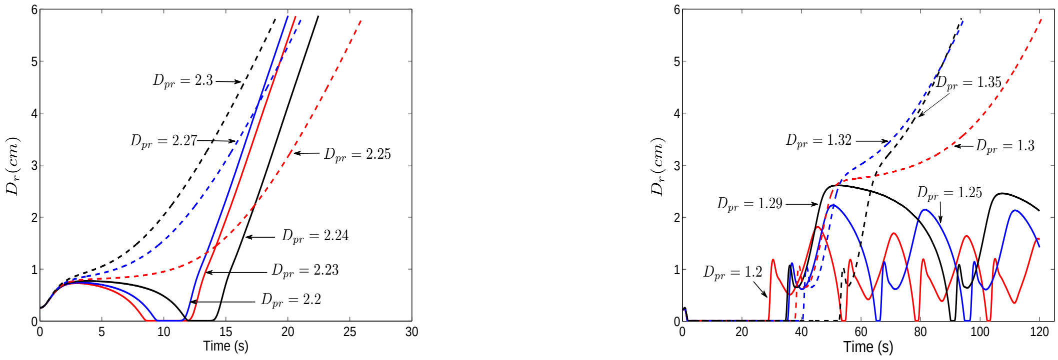

Figure 3. Effect of variations of diameter ratio D_{pr}: Left: $D_{pr} \geq 2$; Right: $1 < D_{pr} < 2$).

As can be seen from figure 2, the one and only DKT process would take place for $D_h/D \leq 2.47$, while beyond this region the two particles continually separate from each other from the start, and thus they can never undergo the DKT process.

Effect of the diameter ratio

As the effect of the diameter ratio on the interactions of two particles is concerned (see figure 3), we can divide D_{pr} into three areas especially for the occurrence of the DKT process: first, the two particles will undergo repeated DKT process at $1 < D_{pr} \leq 1.29$; second, at $1.29 < D_{pr} \leq 2.24$ the two particles will undergo the DKT process only once; third, the DKT process will never occur at $D_{pr} > 2.24$.

CONCLUSIONS

For two particles in which the larger particle is initially located above the smaller one, the increase of the initial longitudinal distance between them have no effect on the occurrence of the characteristic drafting, kissing and tumbling (DKT) process, and the decrease of diameter ratio can enhance the occurrence of the repeated DKT process. While for the case in which the smaller particle is initially above the larger one, the increase of the initial longitudinal distance between two particles yields negative effect on the occurrence of the DKT process, and the results of the effect of diameter ratio demonstrate three transitions in the pattern of dynamics of interaction from repeated DKT process, one-off DKT process and continuous separation from each other.

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