# AN UNIQUE NONLINEAR VIBRATION PHENOMENON OF A LIQUID - ELASTIC SHELL COUPLED SYSTEM

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## Introduction

The dramatic phenomena of mythical antique dragon washbasin have attracted quite a few interests from researchers in mechanic science. Dragon-Washbasin is a brass washbasin with dragon figures carved in the bottom and two handles on the top. With the basin half filled with water, the handles rubbed with hands, myriad of water jets will emanate from the crest of water waves seemed like fountain, see Fig. 1. Experimental studies on both the elastic vessel and the water waves have been carried out by Wang et al [1]. During the vibration experiences on dragon wash, another remarkable phenomenon was observed. When excite the system with a frequency (139Hz) near the resonant frequency (137.5Hz), low frequency (2.5Hz) standing gravity surface was aroused. More wave patterns were observed when doing excitation experiments using cyclic shell partially filled with water instead of dragon wash, see Fig. 2. Literature research revealed that similar phenomenon was observed by Huntley [2] and discussed by J. J. Mahony and R. Smith[3]. D.Y. Hsieh[4] analyzed this phenomenon in some papers. However, previous studies all concentrated on the wave interactions of the fluid domain with dynamic response of the solid vessel assumed to be simple sinusoidal vibration. Whereas Wang's new experimental reveals that vessel vibration has significant modulation when low frequency gravity wave appeared, see Fig.3. This feature demonstrated strong nonlinear coupling of solid vessel and fluid.

Fig. 1 Dragon wash phenomenon

Fig. 2 Low frequency gravity waves in glass shell

In the present paper a new approach is adopted. Other than discussing the nonlinear interaction of waves in the vessel, we study the nonlinear interaction of two natural modes of the solid-fluid coupling system. To establish nonlinear equations, we first studied the variational equations for the liquid-shell coupled system.

## Nonlinear formulations

Variational principle for fluid dynamics in Eulerian description was developed by Luck [5] and Seliger et al. [6]. The application of this variational principle of nonlinear dynamical fluid-solid interaction systems developed by Xing and Price [7] gives a functional describing the shell-water interaction system as follows.

$$\delta J(\phi,\eta,u,v,w)=\delta\int_{t_1}^{t_2}\left(-\iiint_{\Omega}\rho_f\left(\phi_t+\frac{1}{2}\nabla\phi\cdot\nabla\phi+gz\right)dv+T_s(\dot u,\dot v,\dot w)-V_s(u,v,w)\right)dt=\delta W \qquad (1)$$

where $\phi(r,\theta,z)$ is the potential function of the fluid, $\eta$ the height of the free surface, and $u$, $v$, $w$ the displacements of the shell surface in three directions. Applying Rayleigh-Ritz approach, the independent variants, fluid potential function $\phi(r,\theta,z,t)$, free surface height $\eta(r,\theta,t)$, and shell displacement $\mathbf{w}(\theta,z,t)$ can be expanded to the summations of eigenfunctions.

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$$\phi(r,\theta,z,t)=\sum_{i=1}^{\infty}\phi_i(t)\Phi_i(r,\theta,z)\quad \eta(r,\theta,t)=\sum_{i=1}^{\infty}\eta_i(t)H_i(r,\theta)\quad \mathbf{w}(\theta,z,t)=\sum_{i=1}^{\infty}w_i(t)\mathbf{W}_\mathbf{i}(\theta,z)\qquad (2\text{-}4)$$

Substituting (2-4) into the variational equation (1) and doing some mathematical deductions, we get the governing equation with respect to only $\mathbf{q}$ the displacement of the domain surface

$$\frac{d}{dt}(\mathbf{M_f}\dot{\mathbf{q}})+\mathbf{M_s}\ddot{\mathbf{q}}+\mathbf{K_s}\mathbf{q}+\frac{\partial V_f}{\partial \mathbf{q}}-\frac{\partial}{\partial \mathbf{q}}\left(\frac{\mathbf{1}}{\mathbf{2}}\dot{\mathbf{q}}^{\mathbf{T}}\mathbf{M_f}\dot{\mathbf{q}}\right)=\mathbf{Q}\qquad (5)$$

Here, $\mathbf{q}$ denotes the displacement of the domain surface.

$$\mathbf{q}=\begin{Bmatrix}\boldsymbol{\eta}(\mathbf{t})\\ \mathbf{w}(\mathbf{t})\end{Bmatrix}=\{\eta_1\quad \eta_2\quad \cdots\quad w_1\quad w_2\quad \cdots\}^T\qquad (6)$$

$\mathbf{M_f}$ and $V_f$ can be expressed to be nonlinear functions of $\mathbf{q}$ by using Taylor series expansions. Linear part of equation (5) gives the natural modes of the coupled system. And then substituting $\mathbf{q}$ using natural mode coordinates $\mathbf{x}$, we can get nonlinear interaction equations of natural modes $\mathbf{x}$.

## Example

We now apply our nonlinear model into the system under experiment and then compare the theoretical result with experiment data. In experiment we use a glass cylindrical shell 0.005m thick and 0.3m high, with radium of 0.12m. The deep of the water is 0.15m. The material constants for the glass are $\rho_s=2.777\times10^3\,kg/m^3$, $E=10.2\times10^{10}\,N/m^2$. The first symmetric gravity wave of the water has a frequency $\omega_1/2\pi=2.78Hz$ with mode shape described by $J_0(k_{01}r)\cos 0\theta$. And the first resonant natural frequency of the system is $\omega_2/2\pi=242.19Hz$, with corresponding mode form as $\cos 2\theta$. Following the deductions described in previous section, we got the following nonlinear equations of the first two coupled natural modes of the system.

$$\begin{aligned}\ddot{x}_1&=-\beta_1\dot{x}_1-\omega_1^2x_1+F(-12.3664x_1+6.658733x_2)\cos\Omega t+\left(3168.02\cdot x_1^2+2.55817\times10^7\cdot x_2^2\right)\\&\quad+\left(-5.51144\dot{x}_1^2-13.1996\dot{x}_2^2\right)\end{aligned}\qquad (7)$$

$$\begin{aligned}\ddot{x}_2&=-\beta_2\dot{x}_2-\omega_2^2x_2+F(-2.08851x_1+8.84826x_2)\cos\Omega t-8.01584\times10^6x_1x_2+3.57712\dot{x}_1\dot{x}_2\\&\quad-0.583825\cdot F\cos\Omega t\end{aligned}\qquad (8)$$

Comparing theoretical shell displacement curve with experiment result in Figure 3 and 4, we can see that they are in well agreement. Fig. 5 gives the comparison of critical force of multi-scale analysis and numerical calculation for the equations (7-8) with experimental results.

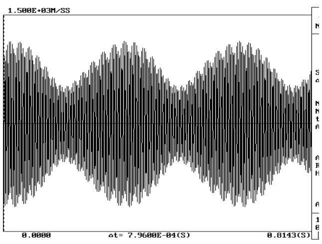

Fig.3 Experimental shell displacement

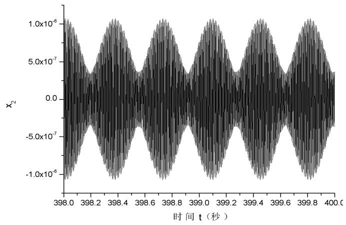

Fig.4 Theoretical shell displacement

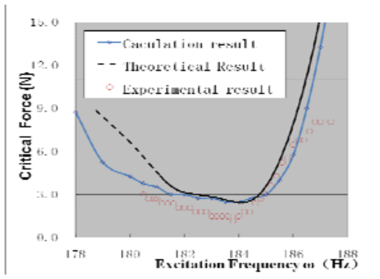


Fig.5 Theoretical and Experimental Critical force

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