FORCED NONLINEAR VIBRATION OF PIPES CONVEYING SUPERCRITICAL FLUID

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Summary Forced nonlinear vibration of viscoelastic pipes conveying supercritical fluid is investigated in the presence of internal resonance. The governing equation is truncated into a perturbed gyroscopic system via the Galerkin method. For the 2-term truncation, the method of multiple scales is developed to determine frequency-amplitude curves in which the softening characteristic becomes the hardening one with the increasing fluid speed. The results are supported by the calculations based on 8-term truncation.

INTRODUCTON

Pipes are commonly used to carry fluids in many engineering installations. Transverse pipe vibration induced by axially flowing fluids has been investigated extensively [1,2]. If the fluid speed is larger than the critical value, the straight pipe equilibrium configuration becomes unstable. The works on pipes conveying supercritical fluid are rather limited.

MATHEMATICAL MODEL AND ITS GALERKIN TRUNCATION

A pipe conveying fluid is hinged to a base undergoing harmonic motion $D\sin\Omega t$ (Fig. 1). The pipe is modeled as an Euler beam with length L, cross-section area A, mass of per length m, area moment of inertia I, and axial tension $\bar{P}$. The pipe material is constituted by the Kelvin relation with Young's modulus E and viscosity coefficient E^*. The fluid, with mass of per length M, moves steadily in a constant velocity U. The gravity is neglected. At axis coordinate x and time t, transverse vibration of the pipe is specified by its neutral axis with transverse displacement $v(x, t)$.

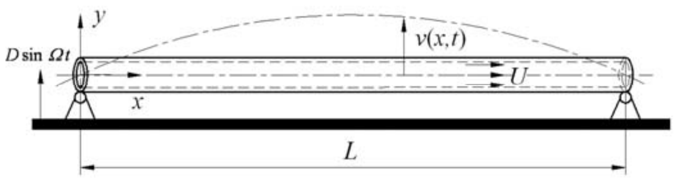

Fig. 1 A horizontal pipe conveying fluid on a harmoniously moving base

The dimensionless governing equation measured from non-trivial equilibrium configurations is

$$\eta,_{\tau\tau}+2M_{\mathrm{r}}u\eta,_{\xi\tau}+\left[u^2-P-\frac{\kappa^2}{2}\int_0^1\left(\eta,_{\xi}+\hat{\eta}_k^{\pm}{}'\right)^2\mathrm{d}\,\xi-\alpha\kappa^2\int_0^1\left(\eta,_{\xi}+\hat{\eta}_k^{\pm}{}'\right)\eta,_{\xi\tau}\,\mathrm{d}\,\xi\right]\eta,_{\xi\xi}+\eta,_{\xi\xi\xi\xi}+\alpha\eta,_{\xi\xi\xi\xi\tau}$$
$$=\frac{\kappa^2}{2}\hat{\eta}_k^{\pm}{}''\int_0^1\left(\eta,_{\xi}^2-2\hat{\eta}_k^{\pm}{}''\eta\right)\mathrm{d}\,\xi+\alpha\kappa^2\hat{\eta}_k^{\pm}{}''\int_0^1\left(\eta,_{\xi}+\hat{\eta}_k^{\pm}{}'\right)\eta,_{\xi\tau}\,\mathrm{d}\,\xi+f\omega^2\sin\omega\tau \tag{1}$$

where

$$\eta=\frac{v}{L},\xi=\frac{x}{L},\ \tau=\frac{t}{L^2}\sqrt{\frac{EI}{M+m}},u=LU\sqrt{\frac{M}{EI}},P=\frac{\bar{P}L^2}{EI},\kappa=L\sqrt{\frac{A}{I}},\alpha=\frac{E^*}{L^2}\sqrt{\frac{I}{(M+m)E}},M_{\mathrm{r}}=\sqrt{\frac{M}{M+m}},\ f=\frac{D}{L},\omega=\Omega L^2\sqrt{\frac{M+m}{EI}} \tag{2}$$

and the non-trivial equilibrium configurations are given by $\hat{\eta}_k^{\pm}(\xi)=\pm\frac{2}{k\pi\kappa}\sqrt{u^2-P-(k\pi)^2}\sin(k\pi\xi)$.

Assume that $\eta(\xi,\tau)$ can be approximated by $\eta(\xi,\tau)=\sum_{j=1}^{n}\phi_j(\xi)q_j(\tau)$, where $\phi_j(\xi)$ (j=1,2,…,n) is the j-th eigenfunction of the free undamped vibration of a beam with the pined-pined boundary conditions and $q_j(\tau)$ the j-th generalized coordinate. Then the n-term Galerkin truncated system is defined by

$$\ddot{q}_1+4M_{\mathrm{r}}u\sum_{j=2}^{n}\frac{1-(-1)^{1+j}}{1-j^2}j\dot{q}_j+\alpha\pi^4\dot{q}_1=\frac{4}{\pi}f\omega^2\sin\omega\tau-\kappa\pi^3\left(\pi\kappa q_1+2\sqrt{u^2-P-\pi^2}\right)\psi\left(q_1,q_2,\cdots,q_n\right)$$
$$\ddot{q}_j+4M_{\mathrm{r}}u\sum_{k=1,k\neq j}^{n}\frac{1-(-1)^{j+k}}{j^2-k^2}kj\dot{q}_k+\pi^4\left(j^4-j^2\right)q_j+\alpha\pi^4j^4\dot{q}_j=\frac{1-(-1)^j}{j\pi}2f\omega^2\sin\omega\tau-j^2\kappa^2\pi^4q_j\psi\left(q_1,q_2,\cdots,q_n\right)\quad(j=2,\cdots,n) \tag{3}$$

where

$$\psi\left(q_1,q_2,\cdots,q_n\right)=\frac{1}{\pi\kappa}\sqrt{u^2-P-\pi^2}\left(q_1+\alpha\dot{q}_1\right)+\frac{1}{4}\sum_{j=1}^{n}j^2q_j^2+\frac{1}{2}\alpha\sum_{j=1}^{n}j^2q_j\dot{q}_j \tag{4}$$

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MULTI-SCALE ANALYSIS BASED ON 2-TERM TRUNCATION

Let n=2 in Eq. (3). To express the smallness of the amplitude of pipe motion and the base motion, as well as the viscosity in the pipe material, they are rescaled as $\eta\leftrightarrow\varepsilon\eta$, $f\leftrightarrow\varepsilon^2 f$ and $\alpha\leftrightarrow\varepsilon\alpha$, where small parameter ε is introduced as a book keeping device. The solutions with the fast and slow time scales $T_0=\tau$ and $T_1=\varepsilon\tau$ is supposed to be

$$q_1(\tau)=q_{11}(T_0,T_1)+\varepsilon q_{12}(T_0,T_1)+\cdots,\ q_2(\tau)=q_{21}(T_0,T_1)+\varepsilon q_{22}(T_0,T_1)+\cdots \tag{5}$$

Then one obtains

$$\mathrm{D}_0^2 q_{11}-G\,\mathrm{D}_0 q_{21}+k_{11}q_{11}=0,\ \mathrm{D}_0^2 q_{21}+G\,\mathrm{D}_0 q_{11}+k_{21}q_{21}=0 \tag{6}$$

$$\begin{aligned}&\mathrm{D}_0^2 q_{12}-G\,\mathrm{D}_0 q_{22}+k_{11}q_{12}=-2\mathrm{D}_1\mathrm{D}_0 q_{11}+G\,\mathrm{D}_1 q_{21}+\alpha_{11}\mathrm{D}_0 q_{11}+\alpha_{12}q_{11}^2+\alpha_{13}q_{21}^2+h\sin(\omega T_0),\\&\mathrm{D}_0^2 q_{22}+G\,\mathrm{D}_0 q_{12}+k_{22}q_{22}=-2\mathrm{D}_1\mathrm{D}_0 q_{21}-G\,\mathrm{D}_1 q_{11}+\alpha_{21}\mathrm{D}_0 q_{21}+\alpha_{22}q_{11}q_{21}\end{aligned} \tag{7}$$

where $\mathrm{D}_0=\partial/\partial T_0$ and $\mathrm{D}_1=\partial/\partial T_1$.

Linear gyroscopic system (6) can be solved to yield the first two natural frequencies ω_1 and ω_2. In the presence of 2:1 internal resonance [3], the first primary resonance is investigated. Introduce the detuning parameters σ_1 and σ_2 to describe the nearness of ω_2 to $2\omega_1$ and ω to ω_1, namely, $\omega_2=2\omega_1+\varepsilon\sigma_1$ and $\omega=\omega_1+\varepsilon\sigma_2$. The solvability condition leads to the steady-state responses in the first and the second modes with the amplitudes a_1 and a_2 satisfying

$$\Lambda_1 a_1^6+\Lambda_2 a_1^4+\Lambda_3 a_1^2-\Lambda_4=0,\ a_2=\frac{|\Gamma_{22}|}{2\sqrt{(\sigma_1-2\sigma_2+\Gamma_{21}^{\mathrm{I}})^2+(\Gamma_{21}^{\mathrm{R}})^2}}a_1^2 \tag{8}$$

Consider a case with M_r=0.447, P=–5, κ=4.0, α=0.001, f=0.1, and ε=0.01. Figure 2 shows the frequency-amplitude curves in the first mode for different detuning parameters dependent on the fluid speed. In the figures, solid (dashed) lines denote the amplitudes of the stable (unstable) steady-state responses and hollow diamonds and solid dots respectively depict numerical solutions of the 2-term truncated system and its modulation equation.

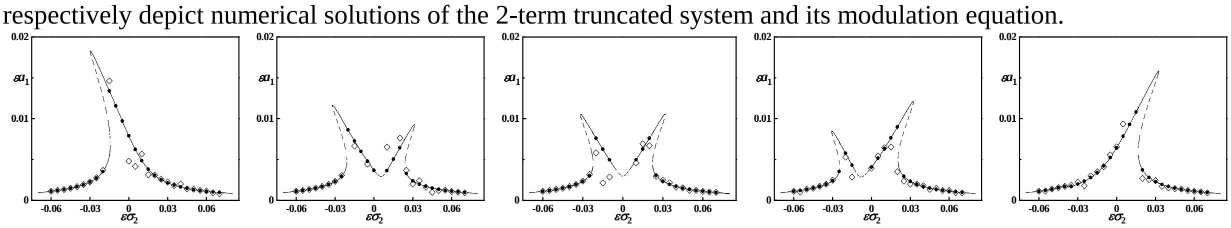

Fig. 2 Frequency-amplitude curves for $\varepsilon\sigma_1$=0.0725, 0.0122, 0.0, –0.0178, –0.0778 (u=5.015, 5.025, 5.02704, 5.030, 5.040)

NUMERCIAL SIMULATIONS BASED ON 8-TRUNCATTION

Let n=8 in Eq. (3) with the same parameters as those in previous section. The resulting ordinary differential equation can be numerically solved. Then the displacement of the pipe centre can be calculated for a given excitation frequency. At different fluid speed, the frequency-amplitude curves are shown in Fig. 3. There are same trends as predicted by the analytical approach. That is, with the increase of the fluid speed, each frequency-amplitude curve is with a peak bending to the left, 2 peaks bending to the opposite directions, and finally a peak bending to the right.

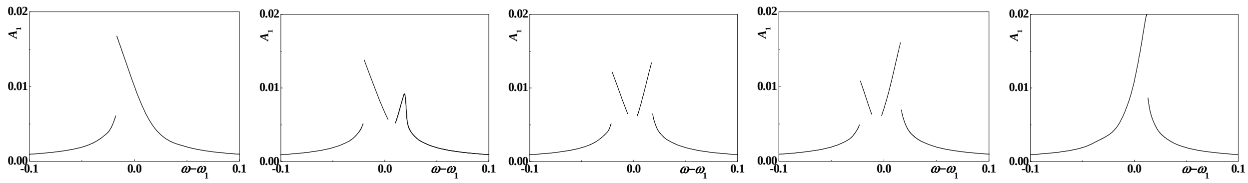

Fig. 3 Stable Frequency-amplitude curves of the pipe center based on 8-term truncation for u=4.82, 4.83, 4.8328, 4.835, 4.84

CONCLUSIONS

Steady-state response in external and internal resonances of pipes conveying fluid is investigated. The fluid flows in the supercritical speed so that the straight equilibrium becomes unstable and bifurcates into two curved equilibria. The method of multiple scales is proposed for two-degree-freedom linear gyroscopic systems with small time-dependent nonlinear perturbations. The investigation on the 2:1 internal resonance and the first primary resonance reveals that the softening characteristic of the frequency-amplitude curve becomes the hardening one with the increase of the fluid speed. The analytical results are supported by the numerical simulations based on 8-term Galerkin truncation.

References

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