DYNAMICS OF RESONANCE OF A FLOATING BODY IN A CHANNEL WITH APPLICATION TO WAVE POWER EXTRACTION

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Summary A new mathematical model is presented to investigate the resonant behaviour of a floating body, namely a thin rectangular flap, inside a channel. When the flap is held fixed in incoming waves, the model allows full representation of the 3D wave field, the free-surface runup and the dynamic torque on the flap. New expressions for the reflection and transmission coefficients are also found. When the flap is free to pitch in incoming waves, the theory shows that such a device behaves as an efficient wave energy converter.

INTRODUCTION

During recent tests on a flap-type structure in a channel [1], peaks in the hydrodynamic actions occurred at certain frequencies of the incident waves. Here this resonant mechanism is shown to be due to the excitation of the transverse sloshing modes of the channel. In the literature, many analytical models are available indeed which investigate the wave field around fixed plates [2, 3]. However, to the authors' knowledge, none of these models offers a comprehensive description of the phenomenon. In particular, the three-dimensional refractive mechanism of the wave field, the reflection and transmission processes at the sides of the plate and their effects on the generation of the excitation torque acting on the plate appear not to have been emphasized enough in previous works. The scenario gets even more vague when considering a moving flap for wave energy conversion. Although much analytical modelling is available for a large number of wave energy converters [4], the analytical treatment of the oscillating flap, also known as oscillating wave surge converter (OWSC), is quite incomplete. An analytical 3D model is developed in the following to fill the gaps in knowledge of this subject, either for a fixed flap (e.g. breakwater) or a moving flap (e.g. OWSC). In both cases, careful application of the Green integral theorem allows to obtain the full analytical expression of the velocity potential and the free-surface elevation at any point of the fluid domain, depending on the geometrical parameters of the system. New expressions are found for the transmission and reflection coefficients of the wave field incident on a fixed flap in the channel that, thanks to the mirroring effect of the sidewalls, also apply to an infinite array of thin breakwaters in the open ocean. The efficiency of the moving flap as a wave energy converter is also assessed. Peaks in efficiency are shown to occur at the resonant frequencies of the sloshing modes in the channel. Parametric analysis is then performed to assess the influence of the main parameter, i.e. the blockage ratio between the flap and the channel width, on the behaviour of the system. Analytical results agree favourably with experimental data.

MATHEMATICAL MODEL

The theoretical basis of the mathematical model is provided by [5] and briefly summarized here. Let the plate be a rectangular box of width w and thickness $2a$, hinged along a straight foundation at a distance c from the bottom of an ocean of constant depth h. The plate is in the middle of a channel of total width b. Monochromatic waves of frequency ω are incoming from the left with wave crests parallel to the flap. A plane reference system of coordinates (x, y, z) is also set, with x on the centre line of the channel, y along the axis of the plate and z positive upwards. Two different problems will be considered, depending whether the flap is fixed or moving, both within the framework of a linear potential-flow theory.

First, consider the flap held fixed in incoming waves. The scattering potential $\Phi^S(x, y, z, t)$ must satisfy the Laplace equation

$$\nabla^2\Phi^S(x, y, z, t) = 0 \tag{1}$$

in the fluid domain, with ∇ the nabla operator. On the free-surface, the kinematic-dynamic boundary condition reads

$$\frac{\partial^2\Phi^S}{\partial t^2} + g\frac{\partial\Phi^S}{\partial z} = 0, \quad z = 0, \tag{2}$$

where g is the acceleration due to gravity. Absence of normal flux at the bottom and through the lateral walls of the channel requires

$$\frac{\partial\Phi^S}{\partial z} = 0, \quad z = -h; \quad \frac{\partial\Phi^S}{\partial y} = 0, \quad y = \pm b \tag{3}$$

respectively. A no-flux boundary condition must be applied on the plate too, so that

$$\frac{\partial\Phi^S}{\partial x} = 0, \quad x = \pm a, |y| < w/2. \tag{4}$$

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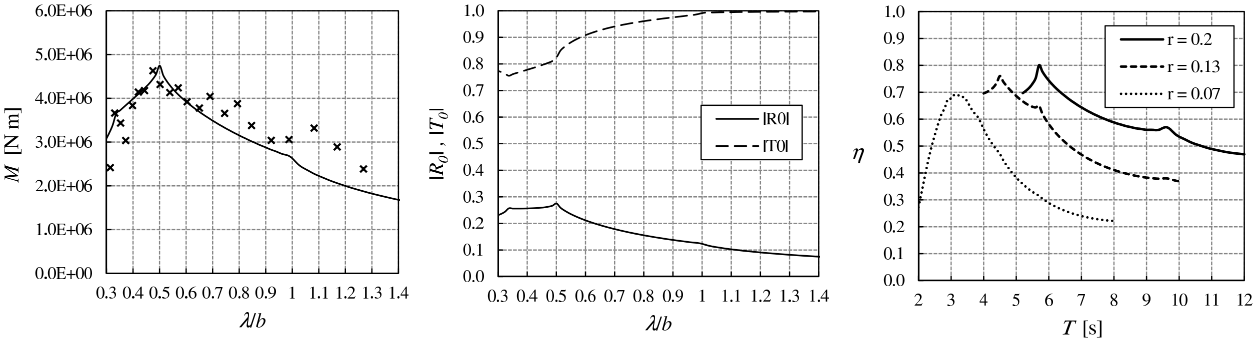

Figure 1. Left panel: torque acting on the fixed flap versus wavelength-to-width ratio. The solid line refers to the analytical results, while the markers refer to experimental data on a flap-type device [1]. Central panel: reflection and transmission coefficients versus wavelength-to-width ratio for a fixed flap. Right panel: efficiency of the flap as OWSC versus period of the incident waves.

Since the total thickness of the flap $2a \ll b$, the thin-plate approximation [6] allows to restate the no-flux condition (4) at $x = \pm 0$. Finally, the reflected and transmitted wave field respectively on the weather side and the lee side of the flap must be both outgoing at large distances from the origin. Application of the Green integral theorem to the governing system of equations (1)–(4) yields an integral equation with a strong kernel singularity. The latter is solved with a new analytical method based on the careful treatment of the singularity [5]. Once the potential is known, the free-surface elevation at any point of the domain and the excitation torque acting on the plate can be found accordingly [5]. Finally, application of the method of stationary phase allows to obtain the asymptotic expansion of the free-surface elevation at large distances from the flap, from which the analytical expressions of the reflection and transmission coefficients can be easily derived.
When the flap is free to oscillate in incoming waves, the velocity potential $\Phi = \Phi^S + \Phi^R$ is given by the sum of the scattering potential Φ^S, found in the previous section, and the radiation potential Φ^R. The latter is the solution of the radiation problem, in which the flap is forced to oscillate with no incident waves. The radiation potential must solve again the governing equations (1)–(3), plus a kinematic boundary condition allowing only tangential motion along the lateral surfaces of the flap

$$\frac{\partial \Phi^R}{\partial x} = -\frac{\partial \theta(t)}{\partial t}\,(z+h-c)\,H(z+h-c), \quad x = \pm 0,\ |y| < w/2, \tag{5}$$

where $\theta(t)$ is the angular rotation of the flap and usage of the Heaviside step function H assures absence of flux through the bottom foundation. This system of equations is solved again with the technique summarized in the previous section. Once the radiation potential is known, the hydrodynamic parameters of the system, i.e. the added mass and the radiation damping, are found accordingly [4, 5]. Finally, the equation of motion of the flap, a forced damped harmonic oscillator, is solved and the efficiency of the device, which depends on the damping coefficient of the generator, is fully characterised.

DISCUSSION AND CONCLUDING REMARKS

Figure 1 shows the behaviour of the excitation torque (left panel) and the reflection and transmission coefficients (central panel) for a fixed flap in the channel, corresponding to the geometry of [5]. The analytical results compare satisfactorily with available experimental data. Note the spiky behaviour of all curves, whose resonant peaks are attained at the wavelength-to-width ratios $\lambda/b = 1/m$, $m = 1, 2, \ldots$. These correspond to the cut-off wavelengths of the sloshing modes in the channel, at which the transverse waves turn from propagating to trapped near the flap. Note also that the larger the reflection coefficient, the larger the excitation torque on the plate. In figure 1 (right panel) the efficiency of the moving flap (OWSC) is also plotted against the period of the incident wave, for several blockage ratios $r = w/b$ of the device. Note that the maximum efficiency is attained at short periods, with the peaks moving towards longer waves while increasing r. For a typical OWSC configuration ($r = 0.2$) the maximum efficiency at resonance is about 0.8 and the average efficiency about 0.6, showing the high effectiveness of the bottom-mounted flap as an OWSC.
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