# EFFICIENT METHODS TO TAKE INTO ACCOUNT THE FLUID LAG IN HYDRAULIC FRACTURING SIMULATIONS THROUGH A VARIATIONAL INEQUALITY FORMULATION

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*Summary* We study the fluid-structure interaction in hydraulic fracturing. Simulating this process needs to take into account the fracturing solid and the fluid flow in the crack. We formulate this coupled problem in which the contact-like condition is accommodated through a variational inequality. The resulted numerical methods save computational cost by *eliminating* the need of: (a) explicitly tracking the fluid front, and (b) switching boundary conditions along different parts of the crack front.

We study the fluid-structure interaction encountered in hydraulic fracturing, a technique used by the oil and gas industry (see [1] and references therein). Simulating this process needs to take into account the interaction between a fracturing solid and the fluid flow in the crack. In particular, with the possible presence of a fluid lag, the evolving fluid front inside the crack and the crack front (crack tip if in 2D) are constrained by a Kuhn-Tucker complementarity condition, which is analogous to the case of contact mechanics (see Figure 1). In particular, let $p \geq 0$ denote the fluid pressure which is a function of the crack face. On, and only on, the subset of the crack face with $p > 0$ (the blue region in Figure 1), the evolution is governed by the Reynolds' lubrication equation. Brute-force methods to keep track of both fronts usually require the use of a costly prediction-correction scheme.

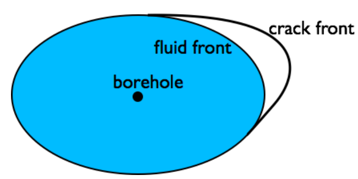


**Figure 1.** Schematic illustrating the different types of boundary conditions required at the crack front. The use of variational inequality eliminates the need of explicitly keeping track of the fluid front and the change of type of boundary condition along the crack front.

In this work, we formulate this coupled problem in which the Kuhn-Tucker condition is accommodated through a parabolic variational inequality with respect to the liquid pressure.
The advantage of this novel methodology roots in the resulted unified formulation for the involved inequality constraints. This allows modeling the evolution of the fluid front with a resolution consistent with the mesh size, and eliminates the need of explicitly tracking the fluid front. Moreover, the need of switching the type of boundary condition along different parts of the crack front is also eliminated. These lead to substantial savings in computational cost.

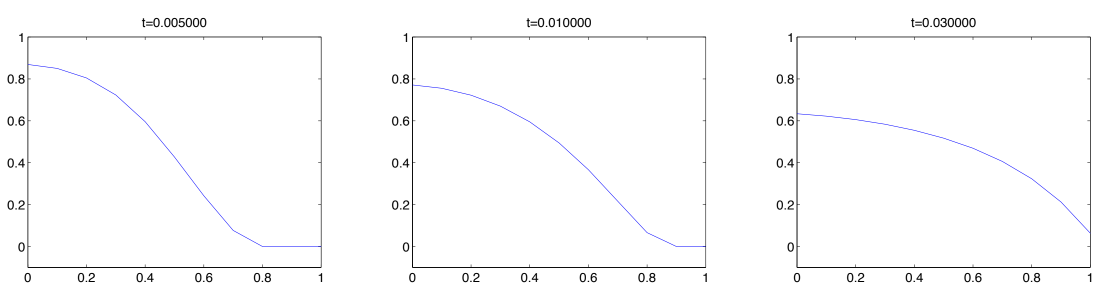


**Figure 2.** Snapshots of the pressure profile ($p$ versus $x$, non-dimensionalized) within a hydraulic fracture ($-1 \leq x \leq 1$) obtained with the variational inequality formulation combined with the displacement discontinuity method. The region with $p = 0$ in the first two subfigures is the fluid lag. We have assumed isotropic linear elasticity in plane strain and symmetry about $x = 0$. The robustness of our method is evident since there is no need of special treatment even there is a change of boundary condition from zero pressure to zero flow at $x = 1$.

We notice that at every time step a discrete variational inequality, instead of an equation, has to be solved in order to

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advance the system. However, solving such an inequality can in turn be recast as a minimization problem, and for this purpose many well-developed algorithms can be adopted from the field of convex optimization, see [2].

We have performed numerical experiments for the model problem of a plane-strain hydraulic fracture in an infinite isotropic medium (Figure 2). In particular, the elastic medium is accounted for by the displacement discontinuity method [3] and the lubrication equation is discretized with the finite element method. The crack is assumed to be non-propagating. This method works as expected: there is no need to keep track of the fluid front location and type of boundary condition; instead, the algorithm automatically switches between different boundary conditions at the crack tip.

Forthcoming work includes the generalization of this method to 3D problems with a moving crack.

## References


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