SH WAVE PROPAGATION IN ONE PIEZOELECTRIC LAMINATE

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Summary The dispersion and transmission coefficients are solved and calculated for the SH wave propagation in one piezoelectric laminate. It is found that the zero-order SH mode is not dispersive and higher mode is dispersive with a cut-off wavelength for the piezoelectricity ignored laminate. It is similar for the electrically open and closed situations, except that the cut-off frequency of the electrically open laminate will shift to relatively high value and the zero-order SH mode of the electrically closed laminate is no more non-dispersive.

INTRODUCTION

Recently, the piezoelectric material was introduced to synthesize special intelligent structures[1-3].The piezoelectricity has changed the constitutive equation, and thus has played a key role in monitoring the wave behaviors. Although the propagation of the relatively complex P-SV wave has been analyzed, the dispersion along with the incident angle is still not enough revealed[3], even for the piezoelectricity ignored case[4]. Thus the SH wave propagation in one piezoelectric laminate is studied to understand the wave behavior.

BASIC FORMULATION AND SOLUTION

The displacement and electrical potential of SH wave are sorted in following forms

$$u_3 = U_3\left(y\right)\exp\left[i\omega(t - s_x x)\right], \quad \varphi = \Phi\left(y\right)\exp\left[i\omega(t - s_x x)\right], \tag{1}$$

with the angular frequency ω, the slowness s_x, the time t and U_3(y) and $\Phi(y)$ the corresponding amplitude. The electrical displacement D_2 and stress σ_{23} satisfy

$$\sigma_{23} = 0, \quad D_2 = 0 \tag{2}$$

at the boundary of y=0 and h for the electrically open laminate, and the dispersion equation of SH wave could be derived as

$$s_x = \pm\sqrt{\frac{\rho}{\bar{c}_{44}} - (\frac{m}{2fh})^2}\ , \ (m=0,\ 1,\ 2,\ 3\ldots) \tag{3}$$

with the mode number m. The elastic constant $\bar{c}_{44} = c_{44} + (e_{15})^2 / \kappa_{11}$ will become c_{44} for the piezoelectricity neglected laminate. The stress σ_{23} and the electrical potential φ satisfy

$$\sigma_{23} = 0, \quad \varphi = 0 \tag{4}$$

at the boundary y=0 and h for the electrically closed laminate, the dispersion equation of SH wave is found to be

$$\left[is_y - \frac{e_{15}^2 s_x}{\bar{c}_{44}\kappa_{11}}(a_{12} - a_{22})\right]^2 - e^{-2is_y h}\left[is_y + \frac{e_{15}^2 s_x}{\bar{c}_{44}\kappa_{11}}(a_{11} - a_{21})\right]^2 = 0\,. \tag{5}$$

The coefficients a_{11}, a_{12}, a_{21} and a_{22} are dependent on the slowness s_x and s_y, and Eq. (5) will become Eq. (3) with $\bar{c}_{44} = c_{44}$ if the piezoelectric effect is neglected.
To solve the transmission coefficients, the displacement of the incident and penetrated wave can be respectively expressed as

$$u_3^{\mathrm{I}} = \left(a^+ e^{i\omega s_y y} + a^- e^{-i\omega s_y y}\right)e^{i\omega(t - s_x x)}, \quad u_3^{\mathrm{II}} = b^+ e^{i\omega s_y (y-L)} e^{i\omega(t - s_x x)}, \tag{6}$$

where $s_y = \sqrt{1/(c_{sh})^2 - s_x^2}$, $c_{sh} = \sqrt{c_{44}/\rho}$. The transmission coefficients can be obtained as

$$\xi = \frac{Z^2 m_{12} + Z(m_{11} - m_{22}) - m_{21}}{Z^2 m_{12} - Z(m_{11} + m_{22}) + m_{21}}, \quad \vartheta = \frac{2Z(m_{12}m_{21} - m_{11}m_{22})}{Z^2 m_{12} - Z(m_{11} + m_{22}) + m_{21}}, \quad Z = \rho_{\mathrm{I}}\,(c_{sh})^2 s_y \tag{7}$$

with m_{ij} the elements of the transfer matrix M of the laminate, and Z the SH wave impedance.

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NUMERICAL RESULTS

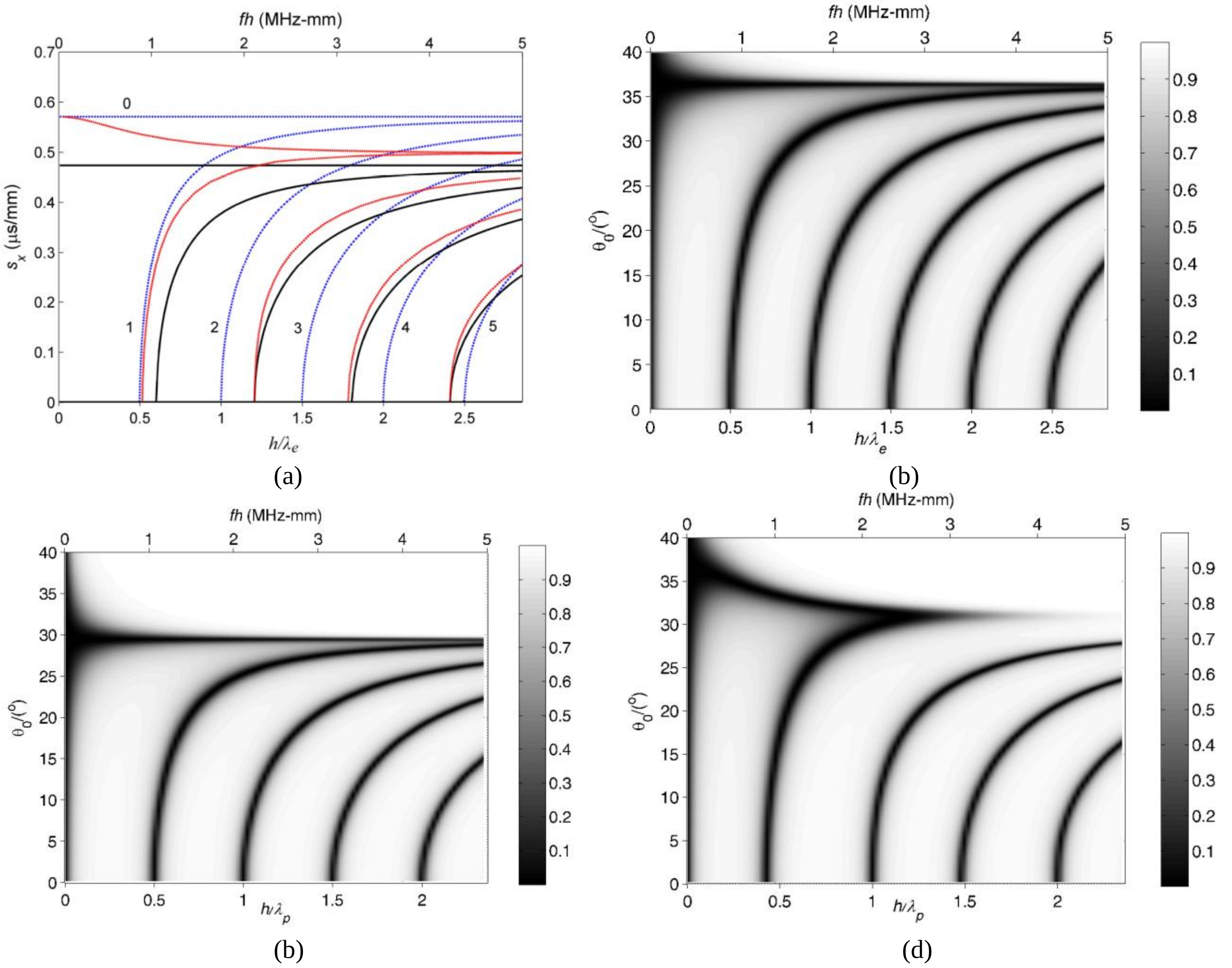

Fig. 1 SH wave dispersion curves (a) and reflective spectrum when piezoelectricity ignored (dot dashed lines) (b), electrically open (solid lines) (c), electrically closed (dashed lines) (d), for one piezoelectric laminate.

The SH wave dispersion curves and reflective spectrum are calculated for the PZT-5 laminate when the piezoelectricity is ignored, or the electrical circuit is open and closed. As shown in Fig. 1, the vertical coordinate corresponds to the slowness s_x or the incident angle θ_0, and the horizontal coordinates respectively correspond to the frequency-thickness product *fh* and normalized wavelength h/λ_e or h/λ_p. Here λ_e is the piezoelectricity ignored wavelength and λ_p is the SH wavelength of the PZT-5 laminate. The reflective spectrum of three cases [Fig. 1(b), 1(c), 1(d)] agree well with the corresponding dispersion curves in Fig. 1(a). For the piezoelectricity ignored laminate, the zero-order SH mode is not dispersive with a maximum slowness, while every higher order SH mode is dispersive with a cut-off wavelength. Same features are found for the dispersion curves of the electrically open and closed situations, except that the cut-off frequency of the former case will shift to the relatively high value and the zero-order SH mode of the latter case is no more non-dispersive. The SH wave can fully penetrate the laminate when there is a corresponding solution in the dispersion curve, and it is fully reflected if there is no solution in the corresponding dispersion curve.

CONCLUSIONS

The dispersion and transmission coefficients are solved and calculated for the SH wave propagation in one piezoelectric laminate. The zero-order SH mode is not dispersive, while every higher mode is dispersive with a cut-off wavelength for the piezoelectricity ignored laminate. It is same for the electrically open and closed situations, except that the cut-off frequency of the electrically open laminate will shift to relatively high value and the zero-order SH mode of the electrically closed laminate is no more non-dispersive.

References

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