THE EFFCT OF ELECTROSTATIC TRACTIONS ON THE FRACTURE BEHAVIOR

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Summary The effect of electrostatic tractions on the fracture behavior of a dielectric material under mechanical and electric loading is analyzed by using a pre-cracked parallel-plate capacitor. Electrostatic tractions produce compressive and tensile stress fields in the material regions, respectively, far away front and behind from the crack tip. Under an applied electric field, there exists a critical mechanical load for crack opening. The crack will open only if applied mechanical load exceeds the critical value. The electrostatic tractions make the crack propagation more difficult due to the mechanical strain energy in the region behind the crack tip.

INTRODUCTION

Electrostatic tractions are usually ignored in linear fracture mechanics for piezoelectric materials so that the mechanical traction-free boundary condition is generally used along crack faces [1-3]. For an electrically insulating crack, electric field exists inside the crack and thus makes the *J*-integral, *J*, different from the energy release rate, *G* [4-6]. Under mechanical and electric loading, induced charges merge along the faces of an electrically insulating crack due to the difference in dielectric constants between piezoelectric material and air (or vacuum) inside the crack. Gao et al. [7] theoretically investigated electrostatic force between the induced charges along crack faces and their influence on the fracture behavior. To make *G*=*J*, Landis [8] proposed the energetically consistent mechanical boundary condition along crack faces in electromechanical materials by using the crack profile after deformation. After that, many researchers [9, 10] re-analyzed the fracture problem of an electric insulating crack in an infinitely large piezoelectric material with the consideration of electrostatic tractions. Recently, Fan el al. [11] discussed in detail the energetically consistent approach and electrostatic tractions along the crack faces and infinite boundaries surrounded by air or vacuum. Their discussions indicate that the energetically consistent approach gives the bifurcation and threshold points of remotely applied stress and electric displacement and between them there are two solutions for crack opening. The electrostatic traction along infinite boundaries is equivalent to an additional mechanical load for a given electric load, which is tensile and promotes crack opening and energy release rate, if the dielectric constant of the infinite medium is smaller than that of the material. Fan et al. [11] made the following statements in the concluding remarks. “In the present work, we assume that both media at infinity and inside the crack be air (or vacuum) so that the electrostatic traction at infinity provides a tensile load. If the medium at infinity has much higher value of dielectric constant than that of the piezoelectric material, the electrostatic traction at infinity will provide a compressive load on the sample, meaning that a higher mechanical load will be required to propagate the crack, which is under investigation.” In experiments and applications, electrodes are usually contacted to piezoelectric or dielectric materials. Electrodes are usually electric conductors with infinitely large dielectric constants. In the present work, we shall consider electrodes and then the effects of electrostatic tractions on the fracture behaviors of dielectric/piezoelectric materials under mechanical and electrical loading.

Energy release rate is one of the most important concepts in fracture mechanics. This is because energy release rate represents the change in a potential energy, which is defined as the applied work minus the internal energy, for crack propagating per unit area. When energy release rate is used as the failure criterion, the critical energy release rate equals twice of the surface energy density for completely brittle cracks and the cracks are called the Griffith cracks. For brittle/ductile solids, the critical energy release rate equals the plastic work plus twice of the surface energy density. To analyze energy release rate for steady crack propagation, a pre-crack strip model is ideal, which considers a semi-infinite long crack in an infinitely long beam. In this case, the energy release rate calculated is completely independent of the crack tip location and determined only by the difference between the potential energy far ahead the crack tip and the potential energy far behind the crack tip. Zhang [12, 13] adopted pre-cracked parallel-plate capacitor models to analyze energy release rate and effects of the pre-crack width and the sample width on the energy release rate. McMeeking [14] developed a pre-cracked parallel-plate capacitor model to evaluate the electric contribution to the energy release rate. To simplify the analysis, the parallel-plate capacitor was held in fixed grips and a fixed free-charge density per unit surface area was applied along the upper and lower surfaces of the capacitor [14]. However, electrostatic tractions were not considered in these analyses [12-14]. In the present work, we develop a pre-cracked parallel-plate capacitor model to study the role of electrostatic tractions in energy release rate. For simplicity, only dielectric materials with isotropic and linear constitutive equations are considered here.

ANALYSIS AND RESULTS

Consider a cracked parallel-plate capacitor. A semi-infinitely long crack is located along the -x axis with the origin of a coordinator system at the crack tip. The parallel-plate capacitor is infinitely large in the x and z directions, but has an original thickness, L_0, in the uncracked part. In the cracked part, each side of the material has an original thickness, l_0, and the crack has an original width, δ_0. The two electrodes are assumed to be mechanically rigid. An electric voltage V and a mechanical displacement ΔL are applied upon the two electrodes. When mechanical displacement ΔL and electric voltage V are independent variables, electric enthalpy, H, will be the appropriate thermodynamics function.

Far ahead the crack tip, i.e., $x \to \infty$, the configuration can be treated as a material parallel-plate capacitor. Far behind the crack tip, i.e., $x \to -\infty$, the material is separated in the middle by a crack gap, which can be considered as three parallel-plate capacitors in series with two material parallel-plate capacitors and one vacuum (or air) parallel-plate capacitor. The two dielectric parallel-plate capacitors have the same thickness of l and the gap between them is δ. Under purely mechanical loading without any applied electric field, $\Delta l = 0$ and $\Delta L = \Delta\delta$, whereas under purely electric field without any mechanical displacement, $\Delta L = 0$ and $\Delta\delta = -2\Delta l$, which indicates that the crack gap will be reduced until two crack faces contact to each other. The electrostatic tractions generate tensile strain $\varepsilon_{yy}^{-\infty} = \Delta l / l_0$ in the material behind the crack tip, which is determined by

$$\frac{(1-\alpha)}{2}\tilde{E}_n^2 = \varepsilon_{yy}^{-\infty}\left(1+\varepsilon_{yy}^{+\infty}-(1-\alpha)\frac{2l_0}{L_0}\left(1+\varepsilon_{yy}^{-\infty}\right)\right)^2, \tag{1}$$

where $\alpha = \kappa_c / \kappa$ is the ratio of the dielectric constant of crack interior to the material one, which is usually smaller than unity, $\tilde{E}_n^2 = E_n^2 / \hat{E}^2$, $E_n = V / L_0$ is the nominal electric field, and $\hat{E} = \sqrt{c / \kappa_c}$ is introduced here for brevity. For a given sample, $\varepsilon_{yy}^{+\infty} = \Delta L / L_0$ represents the applied mechanical load and $\tilde{E}_n$ is the applied electric field in a dimensionless manner. If no electric field is applied, $\varepsilon_{yy}^{-\infty}$ =0 will be obtained from Eq. (1), indicating that mechanical deformation far behind the crack tip is induced by the electrostatic tractions. Once $\varepsilon_{yy}^{-\infty}$ is determined, we can calculate the crack opening displacement. If the original crack gap has a nonzero value, δ_0, the crack opening displacement could be positive or negative, depending on the levels of applied electric and mechanical loads. For example, if only purely electric field is applied, the crack opening displacement is always negative and the absolute value increases with the applied electric field. However, the original crack gap limits the level of the negative crack opening displacement. When the negative crack opening displacement reaches the original crack gap, the two crack faces contact each other and the fracture problem will be converted to a contact problem. The crack closing will be used as a criterion to judge the roots in Eq. (1).

This model finally gives the failure criterion under combined mechanical and electrical loading:

$$\left(\frac{\varepsilon_C^{+\infty}}{\varepsilon_{m,C}^{+\infty}}\right)^2\left[1-\frac{2l_0}{L_0}\left(\frac{\varepsilon^{-\infty}}{\varepsilon_C^{+\infty}}\right)^2\right]-\frac{\tilde{E}_{n,C}^2}{\left(\varepsilon_{m,C}^{+\infty}\right)^2}\frac{(1-\alpha)}{\alpha}\left[(1-\varepsilon_C^{+\infty})+(1+\varepsilon_C^{+\infty})\alpha-\frac{2l_0}{L_0}(1+\varepsilon^{-\infty})\alpha\right]=1, \tag{2}$$

where $\varepsilon_{m,C}^{+\infty}$ denotes the critical applied strain under purely mechanical loading. For a given material, Eq. (2) gives the relationship between the critical applied mechanical load, $\varepsilon_C^{+\infty}$, and the critical electric field, $\tilde{E}_{n,C}$.

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