WAVES IN PYROELECTRIC MATERIALS

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Summary In this paper the governing equation of the thermal electric viscoelastic wave in pyroelectric materials are derived from the inertial entropy theory and the irreversible thermodynamics. The wave speed, wave attenuation etc. are studied by the numerical method. Generally the elastic wave is enhanced by the temperature wave and under certain condition the amplitude of the elastic wave will be increased over time. This phenomenon will be disappeared by given certain viscose resistance in the elastic wave.

BASIC THEORY

In the classical thermoelasticity and thermopiezoelectricity the temperature change does not propagate in a wave form. However the temperature wave at low temperature in the form of heat pulses propagates with a finite velocity. The infinite wave speed and the second sound speed (Landau, 1941) problems induced the development of the generalized heat, thermoelastic and thermopiezoelectric wave theories with a finite phase velocity. The extensive applied theory is the Maxwell-Cattaneo's equation (Joseph and Preziosi, 1989) $q_i + \tau \dot{q}_i = -\lambda T_{,i}$ to instead of the Fourier's law, where τ is a material parameter with the dimension of time. In the thermal elasticity there are mainly three generalized theories. The Lord－Shulman (L-S) theory (1967) applied the Cattaneo's equation, but kept the classical entropy equation $T\dot{s} = \dot{r} - q_{i,i}$. The Green－Lindsay theory (1972) was based on modifying the Clausius-Duhemin inequality and the energy equation： $g = u - \phi s$, $\phi = \phi(T,\dot{T}),\ T = \phi(T,0)$. The inertial entropy theory (IE) (Kuang ,2009) considered that the variation of temperature should be supplied extra heat from the environment and modified the thermodynamic entropy equation by adding a term containing an inertial heat:

$$
\begin{aligned}
&T\dot{s} + T\dot{s}^{(a)} = \dot{r} - q_{i,i} = \dot{r} - (T\dot{\eta}_i)_{,i}, \quad \dot{s}^{(a)} = \rho_s \ddot{T}, \quad s^{(a)} = \rho_s \dot{T} = \tau\left(C/T_0\right)\dot{T} \\
&\dot{s} = \dot{s}^{(r)} + \dot{s}^{(i)}; \quad \dot{s}^{(r)} + \dot{s}^{(a)} = \dot{r}/T - \dot{\eta}_{i,i}; \quad \dot{\boldsymbol{\eta}} = \boldsymbol{q}/T \\
&T\dot{s}^{(i)} = T\dot{s} - T\dot{s}^{(r)} = T\dot{s} + T\dot{s}^{(a)} - \dot{r} + T\dot{\eta}_{i,i} = -\dot{\eta}_i T_{,i} \geq 0; \quad \dot{s}^{(i)} = -\dot{\eta}_i T_{,i}/T
\end{aligned}
\qquad (1)
$$

where T is the temperature, $\boldsymbol{q}$ is the heat flow vector, r is the external heat source strength, s is the entropy saved in the system, $\dot{s}^{(r)}$ and $\dot{s}^{(i)}$ are the reversible and irreversible parts of the s, $T\dot{s}$ is the absorbed heat rate of the system from the environment, $s^{(a)}$ is the reversible inertial entropy, $T\dot{s}^{(a)} = \rho_s T\ddot{T}$ is the inertial heat rate and $\dot{s}^{(a)}$ is proportional to the acceleration of the temperature.; ρ_s is the inertial entropy coefficient, τ is the thermal inertial relaxation time. Comparing Eq. (1) with the classical entropy equation it is found that in Eq. (1) we use $T\dot{s} + T\dot{s}^{(a)}$ to instead of $T\dot{s}$ in the classical theory. According to the irreversible thermodynamics (Groet, 1952; Kuang, 2002) for the viscoelastic problem the stress $\boldsymbol{\sigma}$ is divided into two parts: the reversible part $\boldsymbol{\sigma}^{(r)}$ derived from the electric Gibbs free energy g and the irreversible part $\boldsymbol{\sigma}^{(i)}$ derived from the complement dissipative energy h which is corresponding to the entropy production $\sigma = \dot{s}^{(i)}$. g and h can be assumed respectively in the following simple form (Kuang, 2011)

$$
\begin{aligned}
&g\,(\varepsilon_{kl}, E_k, \vartheta) = u - Ts = (1/2)C_{ijkl}\varepsilon_{ji}\varepsilon_{lk} - e_{kij}E_k\varepsilon_{ij} - (1/2)\in_{ij} E_i E_j - \tau_i E_i \vartheta - \beta_{ij}\varepsilon_{ij}\vartheta - \left(1/2T_0\right)C\vartheta^2 \\
&\delta h = \beta_{ijkl}\dot{\varepsilon}_{ji}\delta\dot{\varepsilon}_{lk} + \eta_j \delta\vartheta_{,j} = \gamma_{ijkl}\dot{\varepsilon}_{ji}\delta\dot{\varepsilon}_{lk} - \left[\int_0^t \left(\lambda_{ij}/T\right)\vartheta_{,i} d\tau\right]\delta\vartheta_{,j}, \quad \vartheta = T - T_0 \\
&C_{ijkl} = C_{jikl} = C_{ijlk} = C_{klij},\ \gamma_{ijkl} = \gamma_{jikl} = \gamma_{ijlk} = \gamma_{klij},\ e_{kij} = e_{kji},\ \in_{kl} = \in_{lk},\ \beta_{ij} = \beta_{ji}
\end{aligned}
\qquad (2)
$$

where u is the internal energy, $\boldsymbol{D}, \boldsymbol{E}$ denote the electric displacement and electric field respectively. $\in_{kl}, e_{kij}, \tau_i, \lambda_{ij}, \beta_{ij}$ C_{klij}, γ_{ijkl} are the material coefficients. The constitutive equations are

$$
\begin{aligned}
&\sigma_{ij}^{(r)} = \partial g/\partial\varepsilon_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \beta_{ij}\vartheta,\ D_i = -\partial g/\partial E_i = \in_{ij} E_j + e_{kij}\varepsilon_{jk} + \tau_i\vartheta,\ s = -\partial g/\partial\vartheta = \beta_{ij}\varepsilon_{ij} + \tau_i E_i + C\vartheta/T_0 \\
&\sigma_{ij}^{(i)} = \partial h/\partial\dot{\varepsilon}_{ij} = \gamma_{ijkl}\dot{\varepsilon}_{kl},\ \eta_i = \partial h/\partial\vartheta_{,i} = -\int_0^t \left(\lambda_{ij}/T\right)\vartheta_{,j}\mathrm{d}\tau,\ T\dot{\eta}_i = q_i = -\lambda_{ij}\vartheta_{,j}; \qquad \sigma_{ij} = \sigma_{ij}^{(r)} + \sigma_{ij}^{(i)}
\end{aligned}
\qquad (3)
$$

From the above equations we get the governing equations for the inertial entropy theory with viscousity (IEV):

$$
T\left(\beta_{ij}\varepsilon_{ij} + \tau_i E_i + C\vartheta/T_0\right)^{\bullet} + TC\left(\tau/T_0\right)\ddot{\vartheta} = \dot{r} + \left(\lambda_{ij}\vartheta_{,j}\right)_{,i} \qquad (4)
$$

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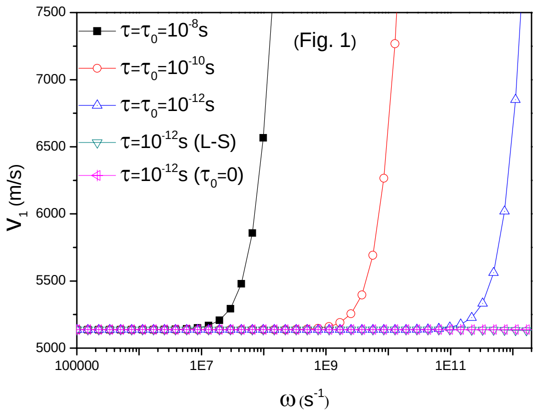
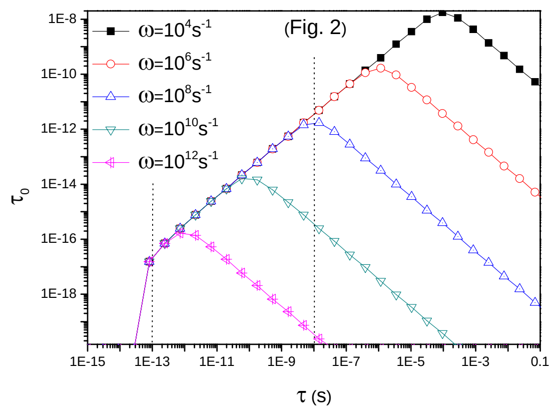
$$\sigma_{ij,j} + f_i = \left(C_{ijkl}\varepsilon_{kl} + \gamma_{ijkl}\dot{\varepsilon}_{kl} - e_{kij}E_k - \beta_{ij}\vartheta\right)_{,j} + f_i = \rho\ddot{u}_i, \quad D_{i,i} = \left(\epsilon_{ij}\ E_j + e_{kij}\varepsilon_{kl} + \tau_i\vartheta\right)_{,i} = \rho_e \tag{5}$$

where ρ is the mass density, ρ_e is the density of the electric charge. Though the viscous effect may also be extended to the electromagnetic field, but for the quasi-static electromagnetic field it may not be needed. Eq. (4) is a temperature wave equation, Eq.（5）is a thermoelectric viscous elastic wave equation. For the pure thermal conductive problem with small change of the temperature Eq.(4) is become to $C\left(\dot{\vartheta} + \tau_s\ddot{\vartheta}\right) = \left(\lambda_{ij}\vartheta_{,j}\right)_{,i}$ which is the same obtained from the Maxwell-Cattaneo's equation. This means that the heat inertia is introduced by heat conduction in this case. By the way if we use the physical variational principle to derive the temperature wave equation (Kuang, 2011), a temperature body force, similar to the Maxwell stress in dielectrics, $\boldsymbol{f}^T = \boldsymbol{\nabla}\cdot\boldsymbol{\sigma}^T, \boldsymbol{\sigma}^T = -(1/2)(s + \boldsymbol{\beta} : \boldsymbol{\varepsilon})\vartheta\boldsymbol{I} \approx -(1/2)s\vartheta\boldsymbol{I}$, can be found.

Examples in an infinite pyroelectric material $\mathrm{BaTio_3}$ for L-S, IE and IEV theories are studied and γ_{ijkl} takes as $\tau_0 C_{ijkl}$, where τ_0 is the viscous relaxation time constant. It is found that the attenuation coefficients and velocities of temperature wave for these three models are almost the same, but for the elastic wave the difference is revealed. Fig. 1 shows that the phase velocities of the quasi-longitudinal wave is nearly equal to the usual elastic wave speed v for $\omega\tau_0 < 0.1$ and increases quickly with the increase of circular frequency for $\omega\tau_0 > 0.1$. Fig. 2 gives the phase figure of attenuation coefficient for various τ, τ_0, in which the attenuation coefficient is positive in the region above the lines and negative in the region below the lines. It is found that if $\tau_0 = 0$ the negative damping is occurred, but for $\tau = 0$ there is no damping region. How to explain and use the negative damping it is also a meaningful problem.



(Fig. 1)



(Fig. 2)

DISCUSSIONS AND CONCLUSIONS

In the L-S theory from the Cattaneo's equation and usual entropy equation we get $T\dot{s} + \tau_0 T\ \ddot{s} = \lambda_{ij}T_{,ji} + \left(\dot{r} + \tau_0\ddot{r}\right)$, so the entropy is not a state function. The second problem is why the viscoelastic term only produced by the heat conduction and the inertial term is appeared? In the G-L theory their method is obviously different with classical thermodynamics. The inertial theory is different with the Maxwell-Cattaneo's equation modifying the Fourier's heat conductive law, rather directly modifies the laws of the thermodynamics by adding the inertial entropy in the entropy equation. The inertial entropy theory is also different with other generalized thermodynamic theory. It is fully consistent with the classical constitutive theory and the irreversible thermodynamics and is the simplest theory to get a temperature wave equation with a finite phase speed. The numerical results are reasonable. We also expect that the inertial entropy theory gets further applications in physics and engineering.

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