SURFACE WAVE PATTERNS: FARADAY WAVES AND CROSS-WAVES

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Summary We summarize results of experimental, numerical, and theoretical investigations of the patterns realized by surface waves in an open rectangular container subjected to combined horizontal and vertical vibrations. Among other results, we show that, despite their name, subharmonic "cross-waves" are not oriented crosswise, but at an intermediate angle with respect to the axis of vibration.

INTRODUCTION

When a fluid interface is large compared to the wavelength of excited modes, distinct patterns may compete with one another to produce complex dynamics. This readily occurs in the vertically forced Faraday system [1], which has been a longtime source of intriguing patterns [2, 3, 4]. Horizontally forced systems also exhibit parametric instability [5], which can lead to so-called "cross-waves", which are localized at the wavemaker (boundary). They are commonly observed in wavemaker experiments [6] and can display complex behavior, including chaotic dynamics and slow modulations resembling solitary waves [7]. There are, however, few cross-wave experiments comparable to the many pattern formation experiments in the vertically forced case (i.e., at relatively high frequencies in large-aspect-ratio containers).

EXPERIMENT

Experiments were performed in transparent square containers 9 cm across inside and 2-5 cm deep, where a sharp step could be used to pin the contact line. A pair of electromagnetic shakers vibrated the container both horizontally and vertically, while images of the fluid surface, lit from below, were captured with a high speed camera. The experiments used DC200 silicone oil (5 cSt, 10 cSt, and 50 cSt) and water, and forcing frequencies between 30 Hz and 100 Hz.
With pure horizontal forcing, localized subharmonic patterns (cross-waves) soon appear with large, apparently perpendicular, ridges near the endwalls. In fact, these ridges are part of a more complex underlying pattern that is rhomboidal, or lozenge-shaped, and shows significant variation in the streamwise direction. A Fourier transform confirms this oblique orientation — about 70° in the case of 50 Hz excitation shown in figure 1.

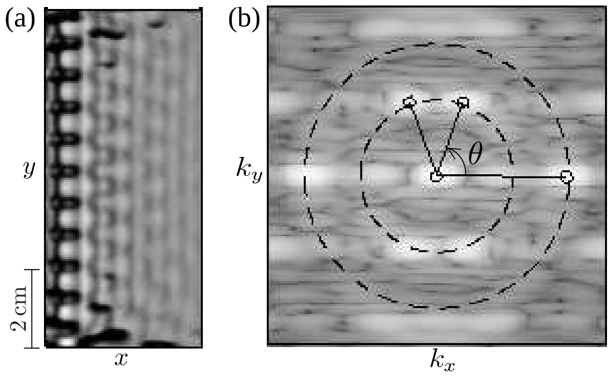

Figure 1. (a) Surface wave pattern and (b) Fourier transform in a 5 cm deep container (pinned boundary) at 50 Hz.

Effect of boundary conditions, depth, and viscosity

Since the parametric forcing is generated by the moving boundaries it is not surprising that boundary conditions play an important role in selecting the surface wave pattern. Critical forcing values are also sensitive to boundary effects since significant damping can occur at the contact line [8].
The surface wave pattern is not very sensitive to variations in fluid depth, at least in the regime where the depth is greater than the wavelength of the subharmonic waves. The primary effect of increasing the fluid depth from 2 cm to 5 cm is to decrease the critical forcing amplitude. This is sensible since a deeper endwall moves more fluid with its motion, leading to greater pressure fluctuations at the free surface. The characteristics of the selected surface wave pattern, though, are largely insensitive to fluid depth.
Similarly, one effect of increasing viscosity is to increase onset values. But, although there is not a lot of difference between patterns observed with 10 cSt oil and 5 cSt oil, the pattern with 50 cSt oil shows a drop in average pattern angle. With water (1 cSt viscosity) the patterns are not easy to analyze due to the strong interaction of the waves generated at opposite walls, which leads quickly to secondary instabilities and spatial disorder.

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Discussion of experimental results

The experimental results establish that, over a wide range of parameters, subharmonic “cross-waves" do not actually orient themselves crosswise to the motion of the endwall or wavemaker. In fact, for the range 30-100 Hz, there is little indication of any transition to ideal “cross-waves" (i.e., to 90°). The qualitative features of the pattern, which reflect the dominance of subharmonic waves oriented at 60°-80° with respect to the forcing, are surprisingly insensitive to changes in wavelength (forcing frequency), fluid depth, or viscosity, although there is noticeable sensitivity to boundary conditions.

NUMERICAL SIMULATIONS

Numerical simulations were performed using a modified version of a parametrically forced surface wave model derived by Zhang and Viñals [9], replacing the uniform (vertical) forcing term with a spatially dependent one. The fact that a reduced model with simplified boundary conditions is in reasonable agreement with the experimental results and yields similar surface wave patterns is an argument for the dominance of the localized forcing mechanism, and for the generic nature of the observed patterns. Furthermore, these patterns seem to be largely determined by the linear problem. That is, while nonlinear effects do come into play, the qualitative character of these patterns is present already in the linear eigenfunctions. This suggests that wavenumber selection in the downstream direction is due to the spatial profile of the parametric forcing mechanism and not the result of resonant interactions between the subharmonic waves themselves.

SIMPLE MODEL WITH NONUNIFORM PARAMETRIC FORCING

A linear two-dimensional system with nonuniform parametric forcing is introduced to describe pattern formation:

$$\ddot{h} - 2\gamma\nabla^2\dot{h} - \nabla^2 h + F(x,t)h = 0, \tag{1}$$

where $h(x,y,t)$ is a scalar field (surface height, say), γ is a dimensionless damping, and $\nabla^2 = \partial_x^2 + \partial_y^2$. The parametric forcing $F(x,t) = f_h g(x)\cos(2t) + f_v \cos(2t+\phi)$ is assumed to be independent of y. We write $h(x,y,t) = \cos(k_y y)[p(x)e^{it} + \bar{p}(x)e^{-it}] + \ldots$, assuming periodic solutions that are standing in the y (crosswise) direction. Substituting into (1) leads to an ODE, parameterized by k_y, that, in general, must be solved numerically. However, the availability of analytical expressions when the forcing is either absent or uniform means that the case of step function forcing, a rough model of a horizontally forced system with forcing only near the boundaries, is especially interesting.

The extent d of the localization is a crucial parameter. When $d \gg 1$ the problem approaches the vertically forced case where solutions may have any orientation. When $d \leq 1$ the problem exhibits the well-known preference for (approximate) cross-waves in horizontally forced systems or wavemaker experiments. As long as the width d is large, boundary conditions play a minor role. In particular, the location of the lowest instability tongue and the pattern associated with its minimum are not significantly affected by the boundaries. As the width d decreases, the critical forcing increases but the minimum (versus k_y) of the instability tongue remains close to $k_y = 1$ and is associated with a cross-wave type pattern with moderate oscillations in the x direction. With small d, the situation is different. The critical forcing with Dirichlet (as compared to Neumann) boundary conditions is considerably higher since requiring $h(0,y,t) = 0$ suppresses the effect of the forcing. In addition, the location of the minimum moves to smaller k_y, which induces more rapid oscillations in x.

The critical forcing required to see patterns in (1) increases with the damping γ. The onset value of k_y, however, remains close to unity until $\gamma \gtrsim 0.1$. At this point the minimum moves toward smaller wavenumbers (which experience less damping) and the relative oscillations in x increase. Viscosity is important in the rotation of the patterns away from ideal “cross-waves”. As $\gamma \to 0$ the critical eigenfunctions oscillate less and less in the x direction,

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