CRITICAL POINT FOR PIPE FLOW, AND BEYOND

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Summary We show that in pipes, turbulence that is transient at low Reynolds numbers becomes sustained at a distinct critical point. Through extensive experiments, simulations, and modeling we were able to identify and characterize the processes ultimately responsible for sustaining turbulence. In contrast to the classical Landau-Ruelle-Takens view that turbulence arises from an increase in temporal complexity, here spatiotemporal intermittency (directed percolation) is the decisive process and intrinsic to the nature of fluid turbulence. The spreading of turbulent domains beyond the critical point is also considered.

INTRODUCTION

More than 125 years ago Osborne Reynolds launched the quantitative study of turbulent transition as he sought to understand the conditions under which fluid flowing through a pipe would be laminar or turbulent. Since laminar and turbulent flow have vastly different drag laws, this question is as important now as it was in Reynolds' day. Reynolds understood how one should define "the real critical value" for the fluid velocity beyond which turbulence can persist indefinitely. He also appreciated the difficulty in obtaining this value. For years this critical Reynolds number, as we now call it, has been the subject of study, controversy, and uncertainty. Now, more than a century after Reynolds pioneering work, we know that the onset of turbulence in shear flows is properly understood as a statistical phase transition. How turbulence first develops in these flows is more closely related to the onset of an infectious disease than to, for example, the onset of oscillation in the flow past a body or the onset of motion in a fluid layer heated from below. Through the statistical analysis of large samples of individual decay and proliferation events, we at last have an accurate estimate of the real critical Reynolds number for the onset of turbulence in pipe flow, and with it, an understanding of the nature of transitional turbulence.

METHODS

For the Re under investigation, any decrease in turbulence fraction results from the spontaneous relaminarization, i.e. decay, of a turbulent puff. Similarly, any increase in turbulence fraction manifests itself in the form of puff splitting, where new puffs are seeded downstream of existing ones. Determining the critical point at which the proliferation of turbulence outweighs its decay and turbulence eventually becomes sustained requires that the time scales of both decay and splitting processes be captured. Moreover, both processes are statistical and require collecting statistics over a large number of samples at each value of Re.

Experiments

Given that turbulence move through the pipe at approximately the mean flow velocity, a long pipe is required to observe long time-scales. Using a precision glass tube with a relatively small diameter (D = 4 ± 0.01 mm) and overall length of 15 m, a total length of 3750 diameters is achieved. A smooth inlet together with careful alignment of the individual pipe sections allows the flow to remain laminar up to Re = 4400. Deviations in Re were kept below ± 5 throughout each set of measurements, which extended over periods of up to 45 hours. This precision was achieved with stringent control of both the pressure difference driving the flow and the water temperature (± 0.05 K).

Direct numerical simulations

To complement experiments and gain insights into the underlying mechanism we also have carried out direct numerical simulations of the Navier-Stokes equations. Two independent numerical codes have been used; one is a spectral-element Fourier code [] and the other a hybrid spectral finite difference code []. Both methods use periodic boundary conditions in the streamwise direction, fix the pipe diameter at $D = 1$, and impose constant volume flux (so that the mean speed $U = 1$), ensuring no variation in Re (which is then given by $1/\nu$), during any run.

Models

In addition to experiments and direct numerical simulations, modeling is used to understand features of the critical point for pipe flow. The models contain two variables, turbulence intensity and mean shear, that evolve according to established properties of transitional turbulence. A discrete model and a stochastic PDE model reproduce almost all large-scale features of transitional pipe flow. In particular they captures metastable localized puffs, puff splitting, and slugs.

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Protocol

The protocol is similar for experiments, simulations, and models. Essentially, puffs are introduced into the flow and then monitored to determine whether they decay or split. From the analysis of large numbers of events it is determined that the decay and splitting processes obey Poisson statistics and are characterized by a mean survival time. The mean survival time is the mean time to decay τ_d or the mean time to split τ_s depending on which of the event type is under consideration.

RESULTS

The critical point

To determine the critical point for the onset of sustained turbulence, we compared the time scale for turbulence to increase through splitting events to the time scale for turbulence to decrease through decay events. The dependence of the mean splitting time τ_s on Re is obtained from a large number of individual experimental and numerical splitting events. All splitting times are well fit with a single super-exponential curve:

$$\tau_s = \exp[\exp(-0.003115Re + 9.161)]$$

Previously measured mean lifetimes for turbulent decay, together with some measurements from the current study, give a single super-exponential fit for mean decay times τ_d as a function of Re

$$\tau_d = \exp[\exp(0.005556Re - 8.499)]$$

The intersection at $Re \approx 2040$ marks where the mean lifetime is equal to the mean splitting time, and to the right of the intersection, splittings outweigh the decay of puffs. Analyzing the data in terms of the turbulent fraction results in the same critical point, confirming the procedure applied here. Moreover, in the model is it possible to compute the equilibrium turbulence fraction (fraction of the flow in the turbulent state) in the long time, long pipe, (i.e. thermodynamic) limit. The turbulence fraction grows continuously from zero at a critical point just above $Re = 2040$. Due to correlations between splitting and decay events, the critical value is not identical to crossing value of Re, but is very close to it (a difference of 0.3%). Just above criticality, the turbulence fraction scale as $\sim (R - R_c)^{0.28}$, supporting that the transition falls into the universality class of directed percolation in 1+1 dimensions.

and beyond

The speed of spreading turbulent domains beyond the critical point is studied. The propagation speed of turbulent-laminar interfaces, i.e. fronts, is determined. The speed of both upstream and downstream fronts is found as a function of Re from just above the critical point to well into the slug region. The upstream front speed various only slightly with Re whereas the downstream front exhibits a number of interesting effects that are well captured by the simple models.

CONCLUSIONS

The "real critical value" for pipe flow in the sense that Reynolds original understood it is obtained for the first time. The critical points corresponds to a directed percolation transition in 1+1 dimensions. The spreading of turbulent domains at higher Reynolds numbers is also considered.

References

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