CONTINOUS AND DISCRET MATERIAL MODELLING AND THE ONSET OF CRITICALITY

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Summary Criticality has been recently recognized as a key indicator of microscopic complexity encountered in crystal plasticity, brittle fracture and damage. The scale free behavior reminiscent of turbulence is revealed in all these rate independent dissipative phenomena through the power law structure of acoustic emission. Recent attempts to understand the origin of self organization towards criticality in solid materials have been based on the study of scalar microscopic models. In this paper we propose a tensorial framework for the modeling of the onset of criticality within a classical engineering theory. The ensuing automata are structurally close to conventional numerical models used in commercial codes. One of the interesting features of the new approach is its inherent size dependence. The model is perfectly adapted for the study of critical behavior in geometrically complex structures subjected to general loading.

Recent experiments (e.g. [3, 1, 2]) suggest that the non-equilibrium steady states associated with plastic deformation in solids are scale-free and therefore critical. Similar observations have been made regarding fracture and damage, however, here we limit our attention to plasticity. Theoretical modeling of criticality associated with plastic deformation has been so far restricted to prototypical scalar models. These models unambiguously confirmed that a relatively simple threshold (or friction) type nonlinearity at the micro-scale combined with long range interactions and quasi-static driving can generate universal critical behavior at the macro-scale. The natural question is how to apply these scalar models to realistic structures subjected to multiaxial loadings.

The objective of this paper is to answer this question and to put forward a first comprehensive tensorial model of criticality in plasticity. In particular, we show that despite many claims to the contrary in the current physics literature, the critical plastic behavior can be studied in the framework of conventional engineering models of plasticity expanded as computational algorithms containing natural length and time scales. The interpretation of these internal scales as genuinely physical parameters requires small but essential modifications in the conventional commercial codes which can be easily implemented. The corresponding techniques which we present here in plasticity setting, can be easily extended to other rate independent dissipative phenomena ranging from magnetism to friction.

The macroscopic phenomenological theory of rate independent plasticity is well established and this theory is known to generate predictions which are in close agreement with macroscopic observations (e.g. [4, 5]). This engineering approach has been recently challenged by the observations of fluctuations revealing scale-free behavior at the microscale. The purpose of this paper is to reconsider the main ingredients of both the phenomenological plasticity and of its computational counterpart in order to show that the mechanisms potentially responsible for the *scale-free behavior* are already there. In our analysis we systematically distinguish between (i) mathematical engineering plasticity and (ii) computational engineering plasticity, or more generally between continuous and discretized description of material behavior. In the mathematical models of rate independent plasticity used by engineers, plastic strain is understood as an internal parameter which evolves when the stress state reaches a yield surface. The plastic strain is a measure of the averaged slip and the averaged incompatibility of the crystal lattice. In particular, this parameter does not see the micro-scale structure of the crystal lattice and presents a lump description of the dislocational phenomena. The goal of the engineering theory is to obtain a fast and accurate assessment of the overall state in the body by circumventing the description of the details of the processes at the scale of the crystal lattice. The resulting mathematical model is known to be highly singular. Computational models essentially regularize the mathematical model by introducing a finite mesh and resolving an iteration-related fast time scale. The computation plasticity model can be viewed as as a finite automaton of a threshold type akin to the ones used in sand pile models. The long-range interaction are described through the elastic stiffness while random thresholds can be implemented through spatial distribution of yield limits. In computational models a small load increment generates an elastic increase of stresses which can then produce a relaxation avalanche leading to stress redistribution over the whole body. We have found that in its conventional form the engineering computational schemes do not generate the observed power law statistics of avalanches. However, the scale free behavior can be recovered within a minimal modification of the commercial codes based on the replacement of the 'optimal' correction algorithm by its 'quantized' analog.

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