ON THE GROWTH AND MORPHOLOGY OF ICICLES

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Summary Icicles form when cool water drips from an overhanging support into air whose temperature is below freezing. Ice growth is controlled by the removal of latent heat which is transferred into the surrounding air via a thin film of water flowing over the ice surface. Predicting the evolving morphology of an icicle is a complex free-boundary growth problem. Theory suggests that overall, icicles have self-similar shapes which are related to those of stalactites. The ice-water interface can also become unstable to form ripple patterns on the surface of icicles. We conducted controlled icicle experiments and analyzed the evolution of the icicle shape and the rippling instability as functions of the ambient temperature, water supply rate, air motion and water purity. We find self-similar shapes under certain conditions. The rippling instability is tied to the purity of the water and ripples climb the icicle during growth.

Icicles are a ubiquitous feature of cold winter weather. The origin of the interesting and diverse morphologies of icicles has only begun to be systematically explored [1, 2, 3]. Icicles are generally harmless, as long as one avoids standing directly beneath them. However, the build-up of ice on power lines [4], aircraft [5] and other structures can be significant hazards. In all cases, ice is formed when latent heat is transferred to the surrounding sub-freezing air from the growing surface. Typically, the liquid water is present only as a thin flowing film over the growing ice-water interface from which the heat is being transferred. On an icicle, this water film is subject to gravitational and viscous forces, as well as to surface tension forces at the water-air interface, and eventually collects in a pendant drop which periodically detaches from the tip. An icicle grows by adding ice on its sides and, by a slightly different mechanism, by tip growth in the neighborhood of the pendant drop [1]. Stalactites has a similar growth mechanism, except that the input water is not consumed and crystallization of $CaCO_3$ is controlled by the release of CO_2 rather than latent heat [6].

While the kinematic growth mechanism of icicles is well-known [1], predicting the emergent shape of an icicle is a non-trivial dynamic free-boundary problem. By coupling the transport of latent heat, the local physics of the water film, and the dynamics of the thermally buoyant boundary layer in the surrounding air, Short, *et al.* made the remarkable prediction that icicles should have self-similar shapes which are related to those of stalactites [2, 6]. In this theory, axisymmetry is assumed and surface tension is neglected. The freezing mechanism at the tip is not treated and tip growth is transformed away. The self-similar shape that emerges is independent of details like the water supply rate, the ambient temperature, and the thermodynamic parameters of water. We performed the first direct test of the self-similar theory of icicle shapes under controlled laboratory conditions.

The surface of an icicle can also become morphologically unstable, leading to the formation of ripple patterns [7]. It has been proposed that this instability, which is presumably related to a similar effect in stalactites [8], crucially involves surface tension effects at the water-air interface [9, 10, 11, 12, 13], and is thus not contained in the aforementioned self-similarity theory for the overall shape [6]. In this paper, we describe observations of these ripple patterns in some laboratory-grown icicles and find that the motion of the ripples is upward during growth, in qualitative agreement with linear stability theory [9, 10, 11, 12, 13].

Studies of the *melting* of icicles [14] have revealed that all forms of heat transport are involved, including advection and diffusion in the surrounding air and water, radiation and the evaporation and condensation of water. Currently, there is no comprehensive theory of the dynamics of icicle growth, incorporating shape, tip growth and ripple formation. In this paper, we describe controlled experiments on the growth of icicles under a variety of conditions. We varied the temperature, input flow rate, water purity, the degree of air motion and the surface tension via surfactants.

We conducted controlled icicle experiments using a table-top icicle growing apparatus, shown in Fig. 1. Icicles were grown in an enclosed refrigerated box with dimensions 38 cm $\times$ 38 cm $\times$ 107 cm in which air motion could be controlled with internal fans. The temperature at the walls of the box was controlled to within $\pm 0.3°$C by a commercial bath which circulated antifreeze through pipes embedded in the walls. The box was insulated with 10 cm of commercial foam insulation. The icicle could be imaged with a computer controlled DSLR camera *via* a slot in the side of the box. The optical path to the camera was insulated, and there were no windows between the camera and the icicle to avoid fogging. The relative humidity inside the box was measured. The icicle was illuminated by a distributed white LED light source against a black cloth background.

The growing icicle hung from a sharpened wooden support that was rotated to encourage axisymmetry and to allow all sides of the icicle to be imaged. The rotation speed was typically 4 min per rotation, which was slow compared to air motions, but rapid compared to the ~ 10 hr growth time of an icicle. The rotational position of the support was indexed to allow the same view of the icicle to be imaged on successive rotations. Water near 0°C was supplied by a nozzle using a peristaltic pump. The temperature of the nozzle was feedback controlled by the computer. Rotation also assisted in the even distribution of water around the icicle. The input nozzle was slightly offset from the rotation axis so that water reached the entire periphery of the support over one rotation. Water that dripped off the icicle was collected by a warmed funnel. Source water types included tap water, distilled water, salinated distilled water and distilled water containing Triton-X, a surfactant.

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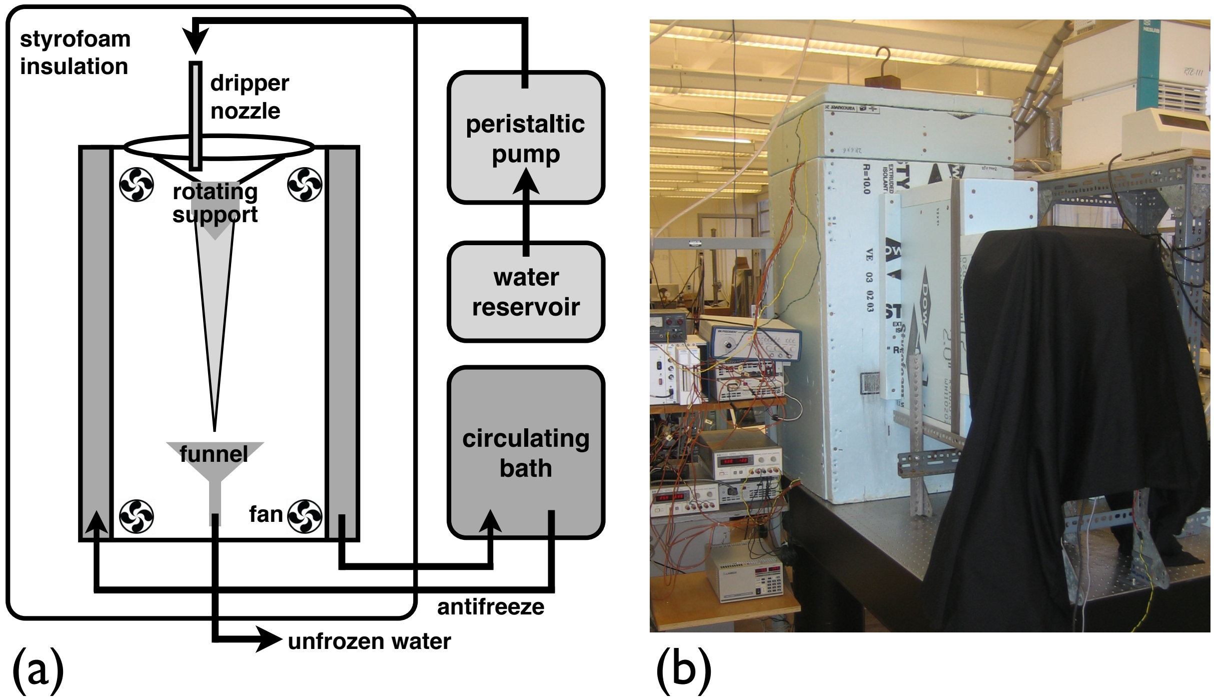

Figure 1. (a) A schematic of the icicle growing apparatus. (b) A photo of the apparatus. The dripper nozzle was temperature controlled just above freezing. The exit funnel was heated and the mass of the output water, and in some experiments its salinity, was continuously measured. The camera is under the black cloth on the right.

Using this apparatus, we grew numerous icicles under varying conditions of air motion, water type, input flow rate, and at ambient wall temperatures ranging between -7°C to -21°C. We measured the shape of the icicles using edge detection from images as well as the mass, and in some experiments the salinity, of the water that dripped off. Results concerning the overall shape and its agreement with the self-similarity theory, are discussed in Ref. [3]. Briefly, we observed that icicles formed from distilled water and stirred air were the most "platonic" — defined to be those closest to predicted self-similar form. Icicles formed from tap water or from saline water, deviated the most strongly from the self-similar shape and were more likely to exhibit clear ripples.

Interestingly, icicles formed with low input flow rates, or under very cold ambient conditions, grow at a constant rate but eventually abruptly stop growing at the tip. This "cessation" occurs because in those limits, it is possible for all the input water to be consumed before reaching the tip. After cessation, the shape of the icicle increasingly deviates from the prediction of the self-similarity theory.

References

[1] L. Makkonen, J. Glaciol. **34**, 116 (1988).
[2] M. B. Short, J. C. Baygents, and R. E. Goldstein, Phys. Fluids **18**, 083101 (2006).
[3] Antony Szu-Han Chen and Stephen W. Morris, Phys. Rev. E, 83, 026307 (2011).
[4] M. Farzaneh (ed.), *Atmospheric Icing of Power Networks* (Springer, Netherlands, 2008).
[5] R. W. Gent, N. P. Dart, and J. T. Cansdale, Phil. Trans. R. Soc. A **358**, 1776 (2000).
[6] M. B. Short, J. C. Baygents, J. W. Beck, D. A. Stone, R. S. Toomey, and R. E. Goldstein, Phys. Rev. Lett., **94**, 018501 (2005).
[7] N. Ogawa and Y. Furukawa, Phys. Rev. E **66**, 041202 (2002).
[8] K. Ueno, S. Yamaguchi, and S. Matsuda (unpublished).
[9] K. Ueno, Phys. Rev. E **68**, 021603 (2003).
[10] K. Ueno, Phys. Rev. E **69**, 051604 (2004).
[11] K. Ueno, Phys. Fluids **19**, 093602 (2007).
[12] K. Ueno and M. Farzaneh, Phys. Fluids **22**, 017102 (2010).
[13] K. Ueno, M. Farzaneh, S. Yamaguchi, and H. Tsuji, Fluid Dyn. Res. **42**, 025508 (2010).
[14] J. A. Neufeld, R. E. Goldstein, and M. G. Worster, J. Fluid Mech. 647, 287 (2010).