TRANSITION TO SUSTAINED TURBULENCE IN PLANE COUETTE FLOW

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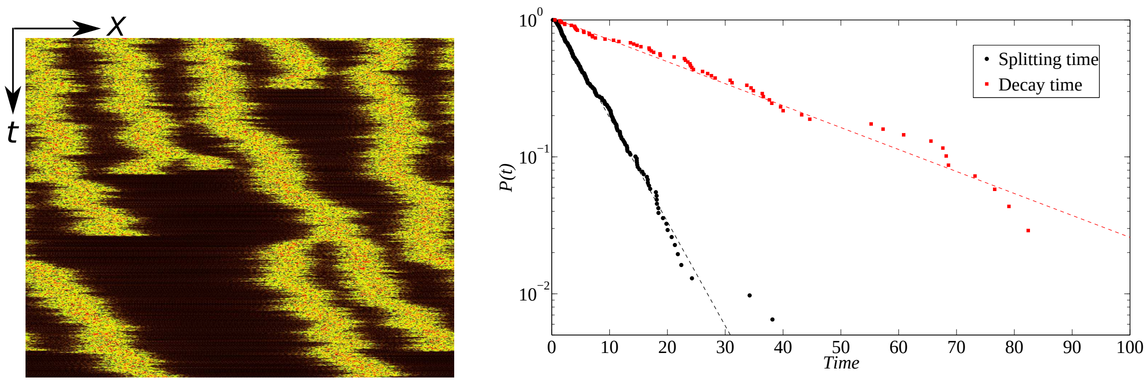
Summary We present here a numerical study on the transition to sustained turbulence in plane Couette flows. At transitional Reynolds numbers, the flow is characterized by large-scale localized turbulent bands surrounded by laminar fluid ([3], [5]). Individual bands are transient in nature but they can also expand and eventually split, overall resulting in complex spatio-temporal dynamics. Here we first investigate the probability distributions of the decay and spreading time of the localized turbulent bands, which determines a lower bound for the critical Reynolds number beyond which turbulence is sustained. Through extensive simulations, the size distributions are further shown to become scale invariant as the Reynolds number approaches the critical point, which is a typical feature of phase transitions. Our results indicate that the transition to sustained turbulence may fall into the directed percolation universality class.

THRESHOLD OF SUSTAINED TURBULENCE

As is reported in [1] and [6], the decay and spreading processes of localized turbulence are key to the sustaining mechanism of intermittent turbulence (see Figure 1(a)). The probabilistic nature of both processes requires statistical approaches to characterize them quantitatively. We here study the probability distributions of the decay and spreading processes of single turbulent bands in plane Couette flow. The Reynolds numbers (based on half gap) under study are in the range of $Re \in [310, 350]$, increased by step 5. At each Re, $N \simeq 300$ realisations with different initial conditions are done. The results are plotted in Figure 1(b), which show the distributions of decay and spreading time at $Re = 330$. We find that at all Reynolods numbers these distributions are exponential and independent of the initial conditions. The dashed lines approximate the data with an exponential function $P^{r,s}(t) = \exp[-(t - t_0)/\tau^{r,s}(Re)]$, where the superscripts (r, s) denote "relaminarization" and "splitting". The characteristic parameter τ is estimated here by the sample mean. The scalings of both $\tau^{r,s}$ with Re are superexponential, indicating the decay and spreading processes of localized turbulenct bands are always possible to happen.



(a) (b)

Figure 1: (a) Spatiotemporal diagram, showing the two dynamical behaviors of localized turbulent bands: decay & spreading via splitting. The colormap shows the streamwise vorticity; (b) Probability distributions of decay and spreading of single turbulent band at $Re = 330$. The dashed lines correspond to the curves $P^{r,s}(t) = \exp[-(t - t_0)/\tau^{r,s}(Re)]$. Y-axis is in logarithmic scale.

In spatially extended systems, spreading of turbulence *via* splitting enhances the turbulent fraction, while its decay eliminates the turbulent fraction. In the thermodynamic limit, if splitting is more probable than decay for each single localized turbulence, the fraction of turbulence in the whole intermittent flow will grow gradually as the time evolves; If these two events happen with equal probability, the flow will reach a dynamical equilibrium state, which is akin to the critical phenomenon of the statistical phase transitions. Hence it makes sense here to compare the timescales τ of these two competing processes. The intersection point of the two curves $\tau^{s,r}(Re)$ fixes a distinct Reynolds number $Re \simeq 325$ where the mean timescales of both processes are in balance, namely, $\tau^s(Re) = \tau^r(Re)$. The value is consistent with the critical value reported in [4] (323 ± 2, experiments) and in [5] (324 ± 1, simulations) in plane Couette flow. Indeed, it gives a lower bound for the critical Re, below which the turbulence decays for sufficiently long observation time.

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SCALE INVARIANCE AT CRITICALITY

We examine thereafter the distributions of the laminar gap sizes between localized turbulent bands. Data are sampled over a time inteval of order $\sim 10^5$ advective time units for each realization. Two different cases are shown in Figure 2: The size distributions follow a power law close to criticality while they are exponential away from the critical point. The observation of scale invariance indicates that the competition between turbulent decay and spreading indeed gives rise to a non-equilibrium phase transition. In order to characterize this transition, we plan to determine the critical exponents at criticality. Based on the data we obtained so far in the regime $Re \in [330, 340]$, we can get a preliminary estimate for the scaling of the turbulent fraction. From a power law fit we obtain an exponent of approximately 0.28 and a critical point around 326.5. An in-depth study is still under way.

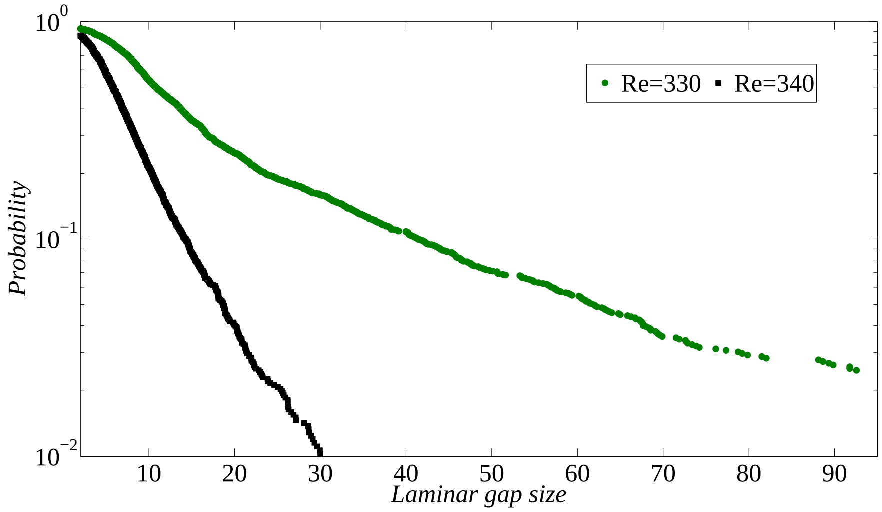

Figure 2: Size distribution of laminar gaps at $Re = 330$ and $Re = 340$. Close to the critical point size distributions become scale invariant (Laminar gaps is distributed as a power law).

CONCLUSIONS

The main objective of this work is to understand the transition to sustained turbulence in plane Couette flows. At moderate Reynolds numbers, localized turbulent states prevail in the whole system and their spatio-temporal dynamics play the key role in the sustainning mechanism of turbulence. The results reported here show that *1)* decay and spreading of localized turbulent bands in plane Couette flow are momeryless and intrinsic to the flow; *2)* the lower bound for the threshold of sustained turbulence is about 325; *3)* the distributions of laminar gap sizes are scale invariant at criticality; *4)* a first estimate of the critical exponent for the turbulent fraction gives a value close to 0.28 which is in excellent agreement with the universal exponent ($\simeq 0.276$) of the 1D directed percolation universality class.

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