FRACTAL CRACKING PATTERN CHARACTERIZATION IN HETEROGENEOUS MATERIALS UNDER COMPRESSION

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Summary In this paper, three different approaches are used to evaluate the fractal dimension of the damage patterns at the peak load and in the post-peak regime of heterogeneous materials subjected to uniaxial compression. The energetic and the statistical approaches are based on the acoustic emission monitoring technique, whereas the renormalization group procedure is based on the assumption of anomalous physical dimensions for the material properties.

APPROACHES FOR CRACKING PATTERN CHARACTERIZATION

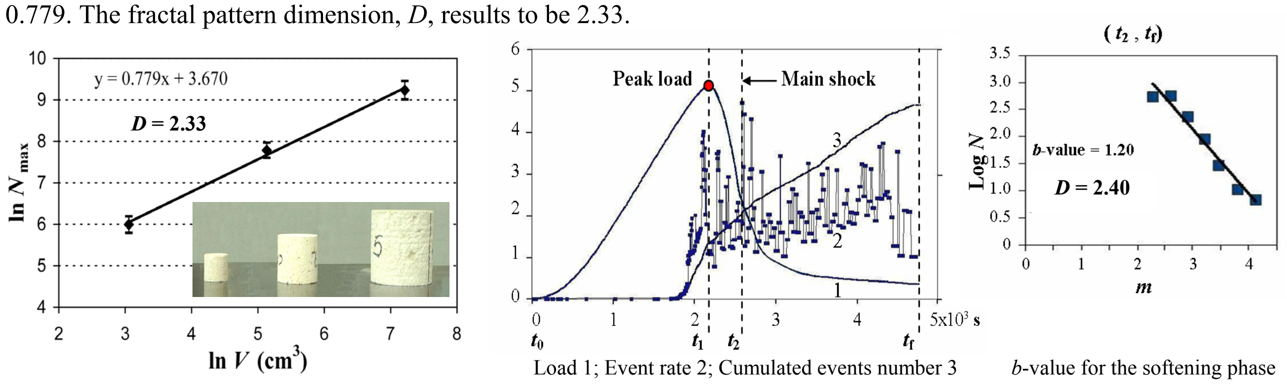
The compressive failure of heterogeneous materials is a complex process governed by damage and fracture. During compression, variations and evolutions of important mechanical parameters of the material take place related to: nucleation, growth and coalescence of microcracks, loss of strength, deformation increments, dissipation of energy. The statistical analysis of a number of this parameters can be performed by the study of the fractal damage pattern evolution during the loading process. This analysis is therefore fundamental for the prediction of a critical state achievement, as well as to determine scale-invariant constitutive laws. In the following three different approaches are outlined.

Energy density approach

Monitoring a structure by means of the acoustic emission (AE) technique, it proves possible to detect the occurrence and evolution of stress-induced cracks characterizing the cracking process. Recent studies have evidenced that the number N of AE events is proportional to the energy release W, that, according to fragmentation theories [1], is related to the specimen volume by means of the following scaling law:

$$W \propto N \propto V^{D/3}, \tag{1}$$

where exponent D is the so-called fractal dimension, comprised between 2 and 3. This means that, not the classical energy density but a fractal energy density (having non-integer physical dimensions) can be considered as the size-independent material property [2]. An experimental validation of this theoretical conjecture is presented in Fig. 1a, where the cumulative AE events number detected during a compression test is reported as a function of the specimen volume (three different cylindrical rock specimens with diameter scaling in the range 1:2:4 and the slenderness equal to 1 have been considered [3]). The parameter $D/3$ in Eq. (1), that represents the slope of the best-fitting curve, is equal to 0.779. The fractal pattern dimension, D, results to be 2.33.



(a) Energy density approach (b) b-value approach

Figure 1. Assessment of the damage pattern dimension: Energy density approach (a); b-value approach (b).

Statistical approach based on the *b*-value

Extensive studies on fracture of brittle materials by means of the AE technique have shown that damage and fracture growth can be characterized through the b-value of the Gutenberg-Richter law. In particular, such a law can be rewritten in order to emphasize a connection between the magnitude m and the size L of the defect associated with an AE event [4, 5]. By analogy with seismic phenomena, the AE crack size-scaling entails the validity of the relationship:

$$N\left(\geq L\right) = cL^{-2b}, \tag{2}$$

where N is the cumulative number of AE events generated by source defects with a characteristic linear dimension $\geq L$, c is a constant of proportionality, and $2b = D$ is the fractal dimension of the damage domain [6]. Therefore, by determining the b-value, that changes continuously in the course of the damage process, it is possible to identify the energy release modalities in a structural element. The extreme cases envisaged by Eq. (2) are D = 3.0, which corresponds to the critical conditions b = 1.5, when the energy release takes place through small defects

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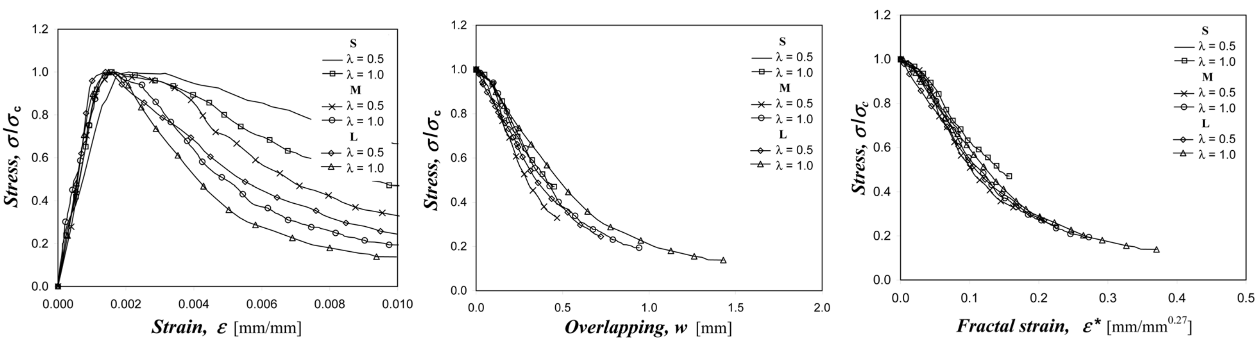
homogeneously distributed throughout the volume, and $D = 2.0$, which corresponds to $b = 1.0$, when energy release takes place on a fracture surface [7, 8]. The analysis of the *b*-value for a compression test of a cylindrical concrete specimen with diameter of 59 mm and slenderness equal to 2.0 is shown in Fig. 1b. The *b*-value, that is the slope of the log-linear AE frequency vs. magnitude diagram, has been evaluated for three different stages during the loading process: a first stage extending from initial time to peak load, a second stage going from peak load to main-shock, as identified by the maximum value of the acoustic emission rate, and a third stage going from main-shock to end of the process. The values obtained for the three stages are 1.64, 1.34, and 1.20, respectively, confirming that the damage pattern approaches dimension 2.0 at the end of the loading process.

Renormalization group theory approach and scale-independent constitutive laws

Damage and strain localization characterising the mechanical behaviour of heterogeneous materials in compression gives rise to size- and slenderness-dependent stress–strain diagrams, with particular regard to the post-peak regime. Such a phenomenon is clearly evidenced by the curves shown in Fig. 2a, referred to concrete specimens with different sizes and slendernesses [9]. In this context, almost scale-invariant constitutive laws can be obtained if the inelastic deformation due to material damage in the softening regime is described by means of a fictitious interpenetration of the material, as represented by the overlapping crack model [9], and shown in Fig. 2b. On the other hand, the assumption of the overlapping crack model that the energy dissipation takes place over a surface is a simplification of a more complex mechanism. A dissipation within a fractal domain is more consistent with the real crushing failure. Accordingly, constant values for the mechanical properties can be obtained only if the classical physical dimensions are abandoned to advantage of noninteger physical dimensions [4, 9]. In particular, an invasive set of fractal dimension $2+d_{\mathcal{G}}$ should be assumed for the energy dissipation pattern, and a lacunar set of dimension $1-d_\varepsilon$ for the deformation distribution inside the specimen. Consequently, the following scaling laws can be derived for the material nominal parameters [4, 10]:

$$\sigma_c = \sigma_c^* b^{-d_\sigma} \; ; \quad \mathcal{G}_F = \mathcal{G}_F^* b^{d_{\mathcal{G}}} \; ; \quad w_{cr} = \varepsilon_{cr}^* b^{(1-d_\varepsilon)} \, , \qquad \text{(3a,b,c)}$$

where σ_y^*, $\mathcal{G}_F^*$ and ε_{cr}^* are the true scale-invariant properties, having anomalous physical dimensions, and *b* is the specimen length. For the considered case, the value of d_ε that optimize the collapse of the curves to a unique fractal overlapping constitutive law is 0.73 (Fig. 2c). The exponent d_σ is assumed equal to zero since the compressive strength presents a statistical dispersion, but not a clear size effect, and, therefore, normalized stresses are considered. Then, the exponent $d_{\mathcal{G}}$ is derived from the relationship $d_{\mathcal{G}} + d_\sigma + d_\varepsilon = 1$, that restricts the maximum degree of disorder [10]. The obtained value, 0.27, reveals that the energy dissipation takes place on a fractal pattern of dimension 2.27.



(a) Stress vs. strain relationship (b) Overlapping law (c) Fractal overlapping law

Figure 2. Compression tests on concrete specimens [8]: Stress–strain diagrams (a); Overlapping laws (b); Fractal overlapping law (c).

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