Study on the chaotic attitude dynamics of the liquid-filled spacecraft by using Hamiltonian-Casimir theory

Baozeng YUE, Wenjun Wu & Salman AHMAD
Department of Mechanics, School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, P.R, China

Summary A canonical Hamiltonian model for liquid sloshing is presented for a moving rigid body. Equivalent mechanical model of pendulum is used to represent the fuel slosh inside the container. Motion stability for the presented model is analyzed using Lyapunov theory with Casimir energy functions. Nonlinear behavior of the system is analyzed under the influence of an external force acting periodically. Chaos is observed through bifurcation diagram.

INTRODUCTION

The movement of fluid inside the moving rigid body can influence the dynamics of object. Such influence may be constructive or destructive for moving vehicle. Free surface movement in moving containers has many engineering applications, such as trucks carrying oil tankers on roads, fluid oscillation inside storage tanks caused by earth quake, sloshing of fluid inside cargo vessels in the ship on sea surface and sloshing of fuel inside the tanks of a spacecraft or airplane. Ibrahim [4] has discussed the detail study about sloshing through experimental data and theory. We are interested to study and control the sloshing inside a moving container for low gravity. There are many parameters that can affect sloshing inside the moving container and hence the stability, such as disturbance frequency and shape of container.

Different dynamical systems such as Hamiltonian and Lagrange dynamics are used for the stability analysis of moving objects. Hamiltonian theory is used for underwater actuation and for some astronaut applications. In our present work, we are using Hamiltonian theory to study and control the dynamics of sloshing inside the tank of a spacecraft. Hamiltonian system is a dynamical system of differential equations that usually formulated in form of Hamiltonian vector fields on a symplectic-manifold or Poisson manifold. The transition to Chaos and Bifurcation for a Hamiltonian system is different than that for dissipative system. For an integrable Hamiltonian system the motion is quasi periodic, i.e. it is oscillatory but depending on more than one independent frequencies. Canonical Hamiltonian Systems include Harmonic oscillator (mass attached with spring or system of couple linear strings), the simple pendulum, some special tops (Euler and Lagrange tops), and the Kepler motion of a planet around sun.

Hamiltonian dynamical system theory is applied on the equivalent mechanical model for sloshing. This equivalent model is used in form of mass attached with spring or a simple pendulum movement [12]. In this research article mass pendulum model is used to derive the Hamilton's equations. Lyapunov theory with Casimir energy function is used to study the stability of the considered model. Casimir energy function has already been used by many researchers in derivation of Lyapunov function for stability analysis [11]. Nonlinear behavior of the system is observed by using bifurcation diagram, time history and phase portrait between angle and angular velocity.

DEVIVATION OF THE SYSTEM EQUATIONS BY USING HAMILTON-CASIMIR THERRY

Consider the elliptical shape of rigid body that represents the spacecraft having translational velocity $\mathbf{v}$ and angular velocity $\mathbf{\Omega}$.

In Fig.1, equivalent mechanical model for sloshing is used having simple pendulum with length l_m attached with a mass m and position vector $\mathbf{r}_m = \langle l_m \sin\theta, -l_m \cos\theta, 0\rangle$ in generalized coordinates system. Fixed mass has been represented by m_F at equivalent distance l_m. Mass of hull of the spacecraft is represented by m_H and the stationary mass with respect to body coordinates is $m_S = m_H + m_F$, hence complete system mass will be $\bar{m} = m_S + m$.

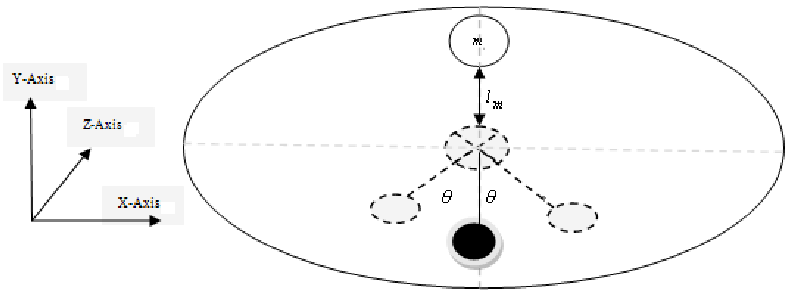

Fig. 1. Equivalent Mechanical Model for Spacecraft Sloshing.

STUDY ON THE ATTITUDE STABILITY AND CHAOS DYNAMICS

We are considering the steady rotation along x-axis. To find the stability of the sloshing equivalent mechanical model, it requires that first derivative of H_Ψ vanishes at equilibrium and second derivative should be positive or negative definite when evaluated at equilibrium (11). For this reason we define, $\Psi_i = \frac{\partial \Psi}{\partial C_i}$ and $\Psi_{ij} = \frac{\partial^2 \Psi}{\partial C_i \partial C_j}$.

First derivative gives us the following results at equilibrium, $\Psi_2^{eq} = 0$ and $\Psi_3^{eq} = -\frac{1}{J_1}$. If steady rotation along x-axis is major axis then $J_1 > J_2$ and $J_1 > J_3$. Second derivative of (11) is positive definite and system can be stable provided,

$l_m \geq (gJ_3)^{1/5}\left(\frac{J_1}{m\Pi_1}\right)^{2/5}$, $\Psi_{23}^{eq} = 0$ and $\Psi_1^{eq} + \Psi_{22}^{eq} = -\Psi_m^{eq}$, where $\Psi_m^{eq} > 0$

Gravitational value of acceleration for low gravity varies from $g = g_1 10^{-6}$ to $g = g_1 10^{-8}$, where g_1 represents the gravitational value of acceleration on the earth's surface [12]. In this research article the value of gravitational acceleration is taken $g = g_1 10^{-6}$.

Positive definiteness of H_Ψ leads to the following propositions for the motion stability,

For major axis steady rotation i.e., $J_1 > J_2$ and $J_1 > J_3$,

(i) $\left[(\bar{m} - m)(J_1 - J_2) + m\bar{m}l_m^2\right]\left[J_2(\bar{m} - m + \Psi_m^{eq}) - ml_m^2\right] > (\bar{m}l_m)^2 J_1$

(ii) $J_1 \geq J_2 - \frac{m\bar{m}l_m^2}{(\bar{m} - m)}$ and $J_2 > \Psi_m^{eq}\left[(\bar{m} - m)J_2 - m\bar{m}l_m^2\right] + ml_m^2$

These propositions provide the conditions for stability of a rigid body carrying fuel container. The sloshing mass of fuel inside the container varies from 10% to 76% of the total mass of spacecraft [15], we assume that sloshing mass is 70% of the total mass, i.e., $m = 0.7\bar{m}$. We use this result to simulate the propositions.

To analyze the nonlinearity of the system, we use Lagrange equation to get equations of motion for the proposed model of pendulum, i.e.,

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] - \frac{\partial L}{\partial \theta} = 0$$

Consider $F_{ext.} = A_d \cos \omega t$, an external force on the system, then the equation of motion for the model becomes;

$$\ddot{\theta} + c_1\dot{\theta} + \frac{1}{2}\sin 2\theta(\Omega_1) = A_d \cos \omega t$$

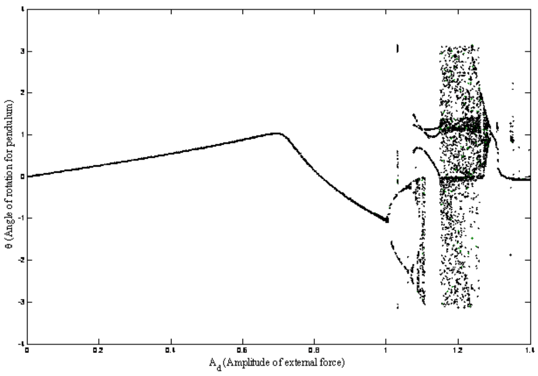

Fig.2. Bifurcation diagram described in figure shows period doubling at $A_d = 1.07$ and becomes chaotic after $A_d = 1.073$ for $c_1 = 0.5$ and $\Omega_1 = 2.0$ .Discontinuity in the bifurcation branch indicates the presence of more than one attractors with overlapping basins of attraction.

where Ω_1 is the rotational velocity component of the rigid body along major axis, A_d is non-dimensional the amplitude of the external force and c_1 is the viscous damping coefficient of the fuel slosh. Simulation results of the nonlinear behavior of the system under the influence of external force are well explained in figures 2.

CONCLUSIONS

In summary, the elliptical shape of rigid body is considered as the spacecraft carrying fuel container that is experiencing sloshing. Hamilton equations are found for the equivalent model of pendulum for sloshing. Internal force is derived for the pendulum model in form of generalized coordinates and equilibrium points are found. Casimir-Energy functions are used for the stability analysis and Lyapunov theory is used to find the stable and unstable regions for $m = 0.7\bar{m}$. It is observed that the mass of slosh and the potential function due to the pendulum movement inside the rigid body have the key roles in the stability of presented model. Relationship for l_m is found which shows its dependence on gravity, moving mass and the angular momentum along major axis. It is obvious that the increase in moving mass results the decrease in pendulum length that justifies the inverse proportionality of l_m with the angular momentum along x-axis inside the fuel tank. Nonlinear behavior of the proposed model is explained graphically under the influence of an external force. Period doubling of the system is observed at $A_d = 1.07$ in bifurcation diagram Fig.3. After the value $A_d = 1.73$, system started to behave chaotically. Chaos behavior of the pendulum model is well expressed through time history map and phase space portrait.

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