EXPERIMENTAL FRINGE ANALYSIS BY COMPRESSIVE SENSING

Howard Y. H. Huang[1, a)] and George Barbastathis[1, 2]
[1]*Singapore-MIT Alliance for Research and Technology (SMART) Centre, Singapore 117543, Singapore*
[2]*Department of Mechanical Engineering, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA*

Summary Experimental mechanics involves large amount of fringe analysis: fringes from laser interferometry such as shearography, electronic speckle pattern interferometry (ESPI) and digital holography, fringes from geometric projection such as shadow moiré and fringe projection, and fringes from photoelasticity. Due to large amount of noise in the experiment data, experimental fringe analysis has posed a difficult task for researchers. In this paper, we develop a series of techniques based on compressive sensing and implement them for noisy experimental fringe analysis. Examples on phase unwrapping, single fringe demodulation and experimental phase optimization will be given.

INTRODUCTION

Fringe analysis is an unavoidable task in a lot of experimental techniques, for example shearography [1], ESPI [2], electron holography [3], shadow moiré [4], fringe projection [5], magnetic resonance imaging [6], interferometric synthetic aperture radar (InSAR) [7] and photoelasticity [8]. The noise in experimental data, in particular the speckle noise by random interference of coherent source, has posed a big obstacle for this task. This situation is even tougher where the fringe density is high or the noise is comparable with phase information.

The noise, however, is not totally random and obeys certain statistical law. On the other hand, the phase to be extracted is related to certain physical quantity and has some known property. Thus experiment result may be optimized by utilizing such information as prior. Compressive sensing [9-10], which utilizes prior such as sparsity or smoothness for optimized information reconstruction from an undersampled linear system, provides a natural way for phase extraction from noisy measurements.

In this paper, we develop a series of techniques based on compressive sensing and demonstrate their capability for phase unwrapping, single fringe demodulation and phase optimization. Preliminary formulation and results will be given in this short paper and more technical details will be included in the presentation.

THOERETICAL BACKGROUND

The problem of experimental fringe analysis stems from the measurement of fringe intensity I which can be expressed as

$$I(x,y) = a(x,y) + b(x,y)\cos[\varphi(x,y)] + N(x,y) \tag{1}$$

where $a(x,y)$ and $b(x,y)$ are the background and modulation terms of the fringe map respectively, $\varphi(x,y)$ is the phase information to be determined and $N(x,y)$ is the noise in certain forms. The task for fringe analysis is to extract the quantitative phase term $\varphi(x,y)$ which is related to the physical quantity to be measured from the intensity measurement I . The cosine function in Eq. (1) has made this inverse problem difficult in two aspects. On one hand, its periodicity has provided the problem multiple solutions, which lead to the necessity of phase unwrapping; on the other hand, the fact that it's an even function has made the problem dual solutions within one period, which lead to the necessity of unambiguous demodulation. The presence of noise $N(x,y)$ further complicates these problems and produces errors in the final reconstructed phase $\varphi(x,y)$.

Compressive sensing [9] tends to reconstruct an optimized estimate x from an undersampled set of measurement y by

$$x = \arg\min_x \{\| y - Ax \|^2 / 2 + \mu\Phi(x)\} \tag{2}$$

where A is a linear operator relating the estimate and measurement, $\Phi(x)$ is a regularization function, *i.e.* the total variation (TV) function or l_0 and l_1 norms, and μ control the relative weight between the fitness term $\| y - Ax \|^2$ and the desirability term $\Phi(x)$. The framework of compressive sensing is closely related to fringe analysis if necessary adaption is applied.

One of the applications of compressive sensing for fringe analysis is path-independent phase unwrapping. By converting a wrapped phase map with 2π phase jumps into a smooth phase gradient map, the noise is amplified and being easier to separate from the slow-variant gradient map. TV minimization or compressive sensing using Fourier transform as operator A for frequency separation can both be implemented for effective denoising without affecting the correct phase gradient values. The correct unwrapped phase can then be obtained by a simple integration. Another

a) Corresponding author. Email: howard.huang@smart.mit.edu.

application of compressive sensing is fringe demodulation. By calculating an ambiguous saw-tooth phase and its gradient from the fringe map, the phase jump areas are then determined and assigned a positive or negative sign to minimize the regularization function $\Phi(x)$. A correct unwrapped phase can then be obtained by integration. In all the applications mentioned above, compressive sensing can be further used for final unwrapped phase optimization. In the optimization, the measurement y in Eq. (2) is now the noise fringe map and the transformation A is now the cosine function linearized at the initial unwrapped phase. The TV function is used as the regularizer $\Phi(x)$ and a denoised unwrapped phase map will then be determined from Eq. (2).

PRELIMINARY EXAMPLE

Due to page limit, we cannot show all the results in this paper. Here we present the path-independent phase unwrapping as an example of compressive sensing and TV minimization. More results will be presented on the conference.
Figure 1 shows a typical setup of shearography which depicts the derivative of out-of-plane displacement. Normally a lot of noise will be presented on the wrapped phase map especially at area with high fringe density which will hinder the phase unwrapping process and produce error in the final unwrapped phase map. The path-independent phase unwrapping method introduced above is applied to such a noisy wrapped phase map in Figure 2(a), and the unwrapping result is given in Figure 2(b).

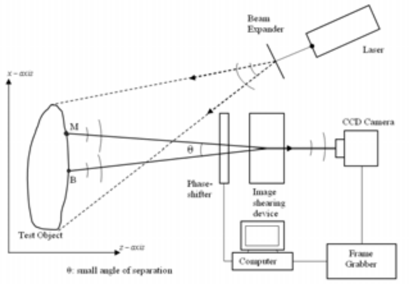
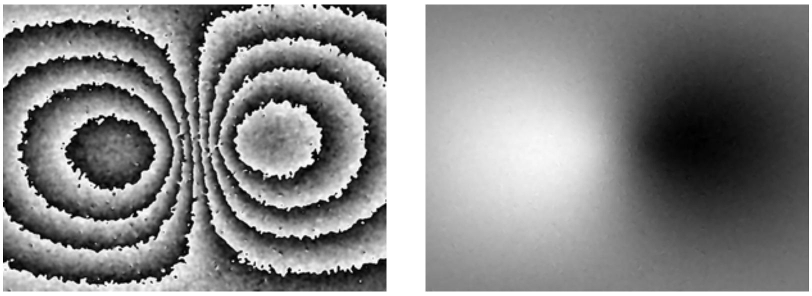


Fig. 1. Schematic experimental setup of shearography for out-of-plane deformation measurement.



(a) (b)

Fig. 2. A typical shearographic wrapped phase map with abundant speckle noise (a) and the unwrapped phase obtained from compressive sensing (b).

CONCLUSION

In this paper, a series of techniques based on compressive sensing have been developed to facilitate experimental fringe analysis. Preliminary result has demonstrated the effectiveness of the proposed methods. With further development and refinement, compressive sensing may provide an effective and systematic tool for experimental fringe analysis in the fields of speckle metrology, InSAR, etc.

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