Experimental investigation of stress concentration for FGMs using CGS

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Summary In this paper, the stress singularity of functionally graded materials (FGMs) under the concentrated load is experimentally studied using Coherent Gradient Sensing (CGS) technology. Firstly, the photo-mechanics equations of CGS for FGM under the concentrated load are established. Secondly, CGS fringe at the vicinity of the concentrated load for FGM specimen are recorded, which indicate the stress singularity of the FGMs. Finally, CGS fringe patterns of FGMs under the concentrated load are simulated, which are in good agreement with experimental results.

INTRODUCTION

FGMs are the non-uniform composite materials with spatial gradient variations of mechanical properties and physical properties, which have widely applications in high temperature structure and biological engineering field.

Nowadays, most studies of fracture mechanics for FGMs are mainly concentrated on theoretical investigation and numerical modelling[1]. However, experimental fracture investigations of FGMs are relatively sparse. CGS is the double grating lateral-shearing interferometric technique, which has the advantage of simple setup, insensitive to external vibrations, non-contact, real-time, full-field optical information, adjustable sensitivity and so on. Marur et al.[2] proposed a gravity sedimentation method to make FGMs using glass microspheres and epoxy. El-Hadek et al.[3] used the reflected CGS to study the dynamic fracture behaviour of FGM material. Yao et al. [4-6] used CGS method to study the basic fracture problems in FGMs, mixed-mode fracture in PMMA material and V-notch fracture. In this paper, CGS method is used to study the stress singularity of FGMs under the concentrated load.

EXPERIMENTAL PRINCIPLE

Fig. 1 shows the geometrical model and the coordinate system of FGM under the concentrated load.The coordinate transformation between the coordinate (x, y) and the coordinate (r, θ) is

$$x = -r \cdot \sin\theta, \quad y = r \cdot \cos\theta . \tag{1}$$

Here, it is assumed that the elastic modulus (E), the density (ρ) and the Poisson's ratio (ν) obey the relationships:

$$E = E_0 \cdot y^k ,\ \rho = \rho_0 \cdot y^k ,\ \nu = \nu_0 , \tag{2}$$

where E_0, ρ_0, ν_0 present the elastic modulus, the density and the Poisson's ratio at the loading point, respectively, k is the gradient coefficients of the elastic modulus and the density.

The photo-mechanics equations of the reflected CGS is

$$\frac{\partial w}{\partial x} = \frac{mp}{2\Delta}, \quad \frac{\partial w}{\partial y} = \frac{np}{2\Delta} \tag{3}$$

where Δ is the distance between two gratings. p is the grating pitch. w is the out-plane displacement. m and n are the fringe order in the x direction and the y direction, respectively. Giannakopoulos et al. [7] proposed the stress distribution for the power function FGMs under the concentrated load. Then the out-plane displacement field of FGMs with the power function under the concentrated load can be expressed as

$$w = \varepsilon_z \cdot \frac{d}{2} = -\frac{\nu \cdot (\tilde{\sigma}_r + \tilde{\sigma}_\theta)}{E} \cdot \frac{d}{2} = \frac{\nu \cdot d \cdot P}{4 I_k \cdot E_0} \cdot \frac{\cos(\beta \cdot \theta)}{r^{k+1}} , \tag{4}$$

where ε_z is the strain perpendicular to the loading plane. d is the thickness of the plate. $\tilde{\sigma}_r$ and $\tilde{\sigma}_\theta$ are the stress components in the r direction and the θ direction, respectively. β is the function of k and ν, which can be expressed as

$$\beta = \sqrt{(k+1) \cdot (1 - \frac{k \cdot \nu}{1-\nu})} . \tag{5}$$

where I_k is the function of k and β [7].

If the main direction of grating is in the x direction, put Eq. (4) into the Eq. (3a). Then the relationship between the fringe order and the applied load is

$$P = -\frac{2E_0 \cdot I_k \cdot m \cdot p \cdot r^{k+2}}{\upsilon \cdot d \cdot \Delta \cdot \left[(k+1)\cos(\beta\theta)\sin\theta + \beta\sin(\beta\theta)\cos\theta \right]} . \tag{7}$$

If the main direction of grating is in the y direction, put Eq. (4) into the Eq. (3b). Then the relationship between the fringe order and the applied load is

$$P = -\frac{2E_0 \cdot I_k \cdot n \cdot p \cdot r^{k+2}}{\upsilon \cdot d \cdot \Delta \cdot \left[(k+1)\cos(\beta\theta)\cos\theta + \beta\sin(\beta\theta)\sin\theta \right]} . \tag{8}$$

EXPERIMENTAL DETAILS

In CGS experiment, the He-Ne laser with 633nm wavelength is selected as the light source. The FGM specimen is non-transparent, and then the optical path of the reflected CGS is set up as Fig. 2. The main direction of grating is perpendicular to the loading direction, which is in the x direction.

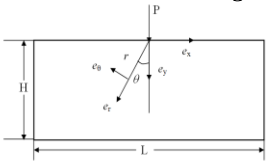

Fig. 1 The geometrical model of FGM

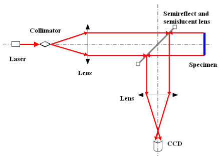

Fig. 2 Optical path diagram of CGS

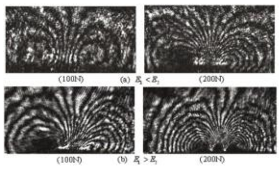

Fig. 3 Experimental fringe patterns of CGS

In this experiment, the grating pitch (p) is 0.025mm, the distance between two gratings (Δ) is selected as 50mm. The specimen (100mm×40mm×6mm) in the experiment is the epoxy/glass microspheres FGMs made by gravity sedimentation method. The elastic modulus in the top (loading point) and bottom of the specimen are assumed as E_1 and E_2. If $E_1<E_2$ ($k>0$), E_1 and E_2 are 3.34GPa and 4.30GPa, respectively, then the elastic modulus of the specimen can be fitted into the power function

$$E = 6.0y^{0.1113}\text{GPa} \qquad (E_1 < E_2,\ \text{k}>0)\,. \tag{9}$$

If $E_1>E_2$ ($k<0$), E_1 and E_2 are 4.30GPa and 3.34GPa, respectively, then the elastic modulus of the specimen can be fitted into the power function

$$E = 2.502y^{-0.1088}\text{GPa} \qquad (E_1 > E_2,\ \text{k}<0)\,. \tag{10}$$

The CGS fringe patterns of FGM specimen under 100N and 200N are shown in Fig. 3(a) and Fig.3(b) for $E_1<E_2$ ($k>0$) *and* $E_1>E_2$ ($k<0$), respectively, which reflect the stress singularity evolution of the FGM specimen.

RESULT AND DISCUSSION

Fig. 4 shows the simulated fringe patterns of CGS of FGM material at 100N for $E_1<E_2$ ($k>0$) and $E_1>E_2$ ($k<0$), respectively, which are agree well with experimental results. With the increasing of the applied load, the fringes gradually expand outward, the radius of the fringes become larger. It will be better to resist the material for the case of $E_1<E_2$ ($k>0$). The simulated load by CGS agrees well with the real applied load. And the error of the CGS method is below 15%, which can accurately reflect the stress singularity of the FGMs.

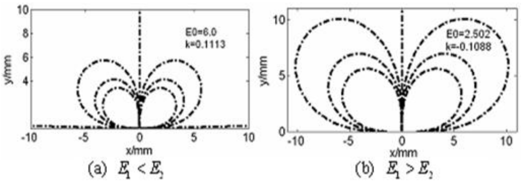

Fig. 4 Simulated fringe patterns of CGS (P=100N)

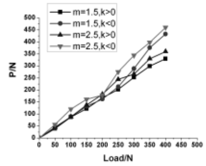

Fig. 5 Comparison between the simulated load by CGS and the real load

CONCLUSIONS

In this paper, the stress singularities of FGMs are investigated by CGS. Some important conclusions are summarized.

(1) The photo-mechanics equations of CGS for FGM material under the concentrated load are established.

(2) The fringe patterns of FGMs under the concentrated load are obtained from CGS experiment and simulation, which are in good agreement with each other.

(3) Based on the CGS fringe distribution, the FGM for $E_1<E_2$ ($k>0$) is more resistance to the applied load than that for the case of $E_1>E_2$ ($k<0$).

(4) The simulated load by CGS agrees well with the real applied load. And the error of the CGS method is below 15%.

Acknowledgments

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