LINEAR DIGITAL IMAGE CORRELATION WITH SOBEL OR SCHARR OPERATORS

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Summary We study the bias and variability in displacement measurements by a linear digital image correlation algorithm which is widely used for optical flow computation. It is shown that a backward version of the algorithm has no noise-induced bias using a noise-free reference image. The bias due to subpixel approximation varies periodically with the magnitude of displacement but can be reduced significantly using a correction procedure. However, the variability in displacement measurements can only be reduced using images with high signal-to-noise ratios and using large subset sizes. The ultimate strain measurement sensitivity using 8-bit grayscale images by this method is also evaluated for thermal-mechanical testing of materials.

INTRODUCTION

Optical flow calculations using Lucas-Kanade gradient-based approach [1] are often implemented using linear gradient filters or operators such as Sobel or Scharr types [2] and their performance is less than optimal due to a significant level of bias [3]. The method can be regarded as a variant of digital image correlation using linear interpolation or simply linear digital image correlation. For precision strain measurement applications for solids, nonlinear digital image correlation is often needed instead where cubic and even quintic interpolation is used for subpixel grayscale approximation [4]. Some recent theoretical analyses and numerical results show that both bias and variability in displacement measurements by digital image correlation with linear interpolation can be eliminated or significantly reduced to a level similar to those obtained by nonlinear methods [5,6]. Here we extend the analysis to linear digital image correlation using Sobel or Scharr gradient operators so effective procedures for bias and variability reduction in displacement measurements can be identified.

1D FORWARD AND BACKWARD LINEAR DIGITAL IMAGE CORRELATION

An image pair of a noise-free reference image $\overline{G}_{0i} \equiv \overline{G}(X_i)$ and its corresponding current image $g_i \equiv \overline{g}_i + \Delta g_i \equiv \overline{g}(x_i) + \Delta g(x_i)$ after some 1D translation *u* (in a fraction of a pixel) are considered in this study [5], where Δg_i is image noise and $0 \le u < 1$. A subset of $i = 1,..., n$ pixel points with integer-valued coordinates X_i in the reference image and the corresponding points with real-valued coordinates x_i in the current image are related by $x_i = X_i + u$. Subscripts for reference or current images with 0, 1, and -1 are used to indicate the image grayscale level evaluated at the whole (integer) pixel points. In 1D cases, both Sobel and Scharr operators are reduced to the common central difference gradient operator. If the image noise $\Delta g(x)$ is white Gaussian noise with a zero mean and a standard deviation σ, one has the expected bias and variance for Δu using the forward central-difference gradient $g_{0i,x} = (g_{1i} - g_{-1i})/2$ as

$$E_{FC}(\Delta u) \equiv E_{FC}^S(\Delta u) + E_{FC}^N(\Delta u) \approx \frac{\sum_{i=1}^{n}\left[\overline{G}_{0i} - \overline{g}_{0i} - \overline{b}_i u\right] g_{0i,x}}{n\left(b_0^2 + \sigma^2/2\right)} - \frac{1}{2b_0^2/\sigma^2 + 1} u, \tag{1a}$$

$$Var_{FC}(\Delta u) \equiv \left[Std_{FC}(\Delta u)\right]^2 \approx \frac{\left[1 + u^2/2\right]\sigma^2 \sum_{i=1}^{n}(\overline{b}_i)^2}{n^2\left(b_0^2 + \sigma^2/2\right)^2} = \frac{\left(2 + u^2\right)}{n} \frac{2b_0^2/\sigma^2}{\left(2b_0^2/\sigma^2 + 1\right)^2}, \tag{1b}$$

where $b_0^2 = \sum_{i=1}^{n}(\overline{b}_i)^2/n$. One can in principle reduces the variability in Δu to be negligibly small by using a large subset or block size *n* but both *subpixel approximation bias* E_{FC}^S and *noise-induced bias* E_{FC}^N in displacement estimation will be unaffected. Similar to [5], a backward version of the method can be implemented and one has the expected bias and variance for Δu as (letting $B_i = \left(\overline{G}_{1i} - \overline{G}_{-1i}\right)/2$ and $B_0^2 = \sum_{i=1}^{n} B_i^2/n$)

$$E_{BC}(\Delta u) \approx -\frac{\sum_{i=1}^{n}\left[\Delta\overline{G}_i\right]_S \overline{B}_i}{nB_0^2}, \qquad Var_{BC}(\Delta u) \equiv \left[Std(\Delta u)\right]^2 \approx \frac{1}{n}\frac{\sigma^2}{B_0^2}, \tag{2}$$

where $\left[\Delta\overline{G}_i\right]_S = \overline{g}_{0i} - \overline{G}_{0i} + (\overline{G}_{1i} - \overline{G}_{-1i})u/2$ is the grayscale error due to subpixel approximation. It is noted according to Eq.(2) that there would be only *noiseless* subpixel approximation bias but no *noise-induced bias* at all in the

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backward linear digital image correlation with a noise-free reference image and a current image with additive white Gaussian noise.

RESULTS

Representative numerical results based on a set of synthetic noiseless images are shown in Fig.1 for both backward-difference gradient (linear interpolation) and central-difference Sobel or Scharr gradient. Details on generating both noiseless and noisy synthetic image sets used in the numerical analyses have already been in given in [4,5]. The maximum subpixel approximation bias for the central-difference gradient (such as the first term in Eq.1a) is about 4 times of that for the forward-difference gradient (0.084 pixels vs. 0.019 pixels). When the forward analyses were carried out with an either forward-difference or central-difference gradient for noisy image sets, the total error in displacement estimation included the noise-induced bias as well. The overall maximum error is comparable for both methods for the noisy image set with a noise level of 4%. The linear dependence of the noise-induced bias on the displacement increment *u* with a negative slope (see 2nd term in Eq.1a) using a central-difference gradient in a forward digital image correlation analysis was confirmed by the numerical results of noisy synthetic image sets. However, there existed no noise-induced bias but only subpixel approximation bias in a backward digital image correlation analysis using a noise-free reference image (see Eq.2). Because the periodic nature of the bias due to subpixel approximation, one can apply the correction procedure first suggested in [5] to reduce the bias level, see Fig.2 and Fig.3. If only ±1 pixel offsets are used, the bias is reduced by about ten times to 0.0084 pixels. If additional offsets of ±0.5 pixel offsets are also used in the correction step, the final maximum bias due to subpixel approximation is about 0.0015 pixels for a noiseless image pair. For a given noisy current image (but a noiseless reference image), the variability in displacement measurement can only be reduced by using a large subset (a very large *n* in Eq.2).

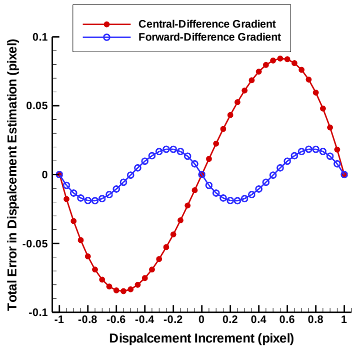

Fig.1 The subpixel approximation bias.

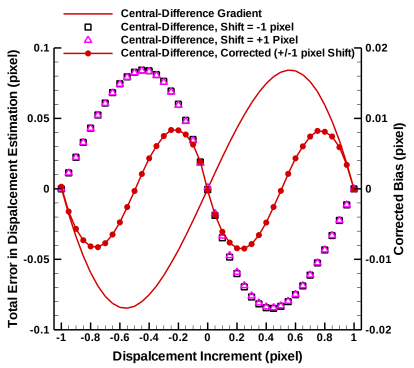

Fig.2 Bias reduction by ±1 pixel offsets.

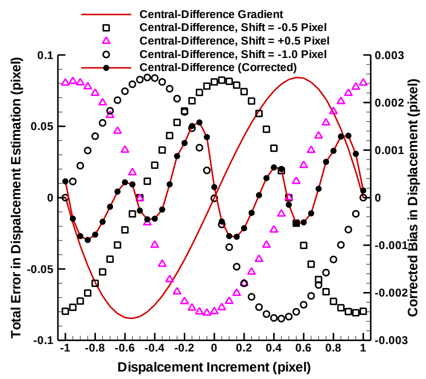

Fig.3 Bias reduction by ±0.5 pixel offsets.

CONCLUSIONS

A backward linear digital image correlation analysis using a noiseless reference image produces no noise-induced bias even when a Sobel or Scharr operator is used. Subpixel approximation bias can be reduced to a level of about ±0.001 pixels using four different offsets of the reference image. When the variability level is also reduced to the similar level using large subsets, the expected strain measurement sensitivity over a gage length of 500 pixels would be about 0.002/500 or 4×10^{-6}. Such sensitivity will be suitable for thermal-mechanical characterization of materials due to thermal expansion or elastic residual stress.

References

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