3-D SHAPE RECONSTRUCTION OF MICRO-VARYING LIQUID SURFACE

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Summary A novel method based upon transmission orthogonal grating were proposed for3-D shape reconstruction of micro-varying liquid surface. By measuring grating deformation under the transparent liquid, in-plane full field displacement vector of the grating was evaluated, which is directly associates the 3-D shape of deformed liquid surface. Theories and equations for the methods are thoroughly delivered. Validation test to measure the deformed water surface caused by a Chinese 1-cent coin has been conducted. The obtained results show that the method is robust in 3-D shape reconstruction of micro-varying liquid surface with a sub-micron resolution.

INTRODUCTION

Disturbances like boundaries or floating bodies will cause the surface to assume a shape different from a horizontal plane, i.e. a deformed liquid surface assumes a necessary shape determined by the equilibrium of gravitational and surface tension and buoyant forces. Such deformations are determined largely by surface properties of the fluid and the floating body itself. The deformed liquid surface associates the physical and chemical reactions between the solids and the fluids, and the biological kinematics of aquatic animal in the gas-liquid interface etc., thus to accurately determine the 3-D shape of a deformed liquid surface is of practical significance.

It can always be found in nature that many aquatic insects can slip through the water. Water striders have remarkable water-repellent legs that enable them to stand effortlessly and move quickly on water, a feature believed to be due to a surface-tension effect caused by the special hierarchical structure of the legs. Even some light metals which density is larger than water can float on the liquid surface. David L Hu et al [1] have designed a mechanical water strider that can float and rowing on the water surface without breaking it. Their study renders, from the literature review, that the balance is maintained between the gravity and surface-tension, which is supported by the deformed liquid surface and numerical simulated by Xiqiao Feng et al, see literature [2]. Since experimental studies are hardly available, detailed floating mechanism of solids still returns existence controversy.

This paper is intended to develop a novel method for measuring micro 3-D shape of deformed liquid surface based on use of metrological grating technique, with the light transmittance and refractivity of the liquid been fully exploited. Mathematical relationships between the out-of-plane deformation of liquid surface and in-plane displacement vector of transmission-virtual grating are theoretical derived.

Basic principle of the developed method

A horizontal liquid plane can subside to form a 3-D shape without breaking the water surface, which is modulated by the gravity of floating solid and supported by the surface tension. This out-of-plane deformation of liquid surface will lead to changes of the exit angle of transmission light that come out from the water, and finally result in a deformation of the virtual image of the object submerged in water. When a periodic grating is placed in water, the transmission-virtual grating image will modulated by the 3-D shape of the liquid, the in-plane deformation of grating is directly related to the out-of-plane deformation of the liquid surface. By physically quantifying the two deformations, the 3-D shape of the deformed liquid surface can be revealed.

Figure 1(a) shows a simple geometrical shape of deformed liquid surface induced by a floating object for the analysis. The out-of-plane displacement $\Sigma h_i(x,y)$ is measured relative to the horizontal liquid plane indicated on the figure. The transmission-virtual grating has a spatial migration (A→A') due to the changing of emission light and finally projects to an image plane as seen in the detector (A'→A''). The observed in-plane displacement of transmission-virtual grating (A→A'', see figure 1(c)), and the relationship between the 3-D shape and the in-plane displacement can be expressed in formula(1):

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$$\begin{cases} \tan(\beta_i - \alpha_i) = s(x,y)/(H - \sum h_{i-1}) \\ \tan\beta_i = dh/ds \\ \sin\beta_i = n \cdot \sin\alpha_i \\ s(x,y) = u(x,y)\cdot i + v(x,y)\cdot j \end{cases} \tag{1}$$

where i is the split-step related to the computing precision, i = 1,2,3…; the water height H is assumed as a given quantity; β and α are the incident and reflection angles, determined by the nature of the liquid, n respects the refraction index of liquid. From Eqs. (1), the step height $h_i(x,y)$ can be reverse derived by Newton iteration method, with the out-of-plane displacement $\sum h_i(x,y)$ then be evaluated step by step.

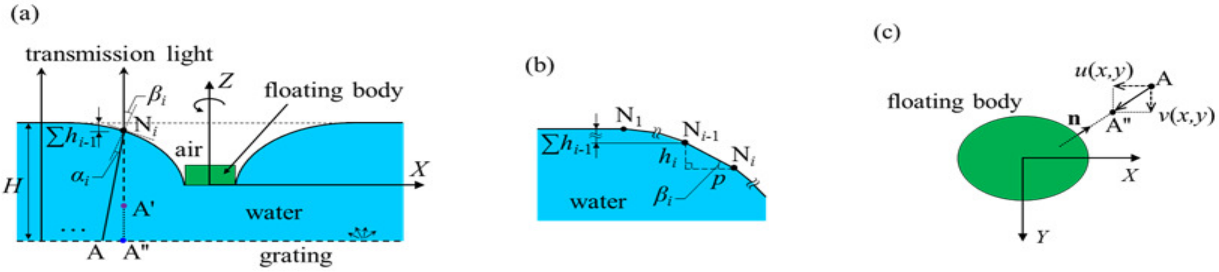

Figure 1. Schematic diagram of the deformed liquid surface induced by a floating body. (a) Cross-section view of the deformed liquid surface. (b) Partially magnified of (a). (c) Displacement of point A to A" (A→A") along **n** direction in the recorded image plane caused by the deformed liquid surface.

Validation test

In daily life it can always be found that some light metals (such as coin, pin and paper clip) can float on the water surface even its density is much larger than the water. Static equilibrium only from perspective of gravity and buoyancy force is restrictive. The liquid surface deformed along the boundary of the floating body to create surface tension used for the physical balance. Here, this out-of-plane deformation of liquid surface caused by a Chinese 1-cent coin is intended to be measured by the developed method to demonstrate its ability. By directly substituting the obtained displacement field to Eq. (1) with applying Newton iteration method, numerical solution of all sequence of points can be computed. Figure 2 (a) shows an arbitrary cross section of the water surface crossing the center of the coin, and figure 5 (b) shows the revealed full-field 3-D shape of deformed liquid surface.

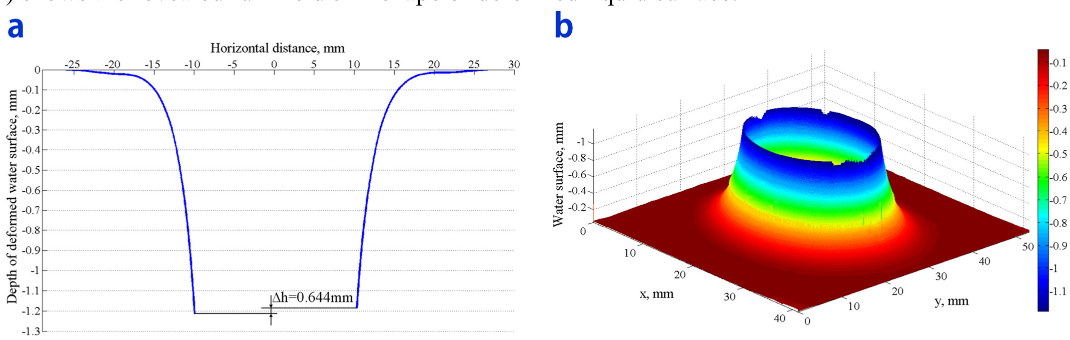

Figure 2. Deformed water surface: (a) a cross section of the water surface; (b) 3-D shape of liquid surface.

CONCLUSIONS

A simple 3-D shape reconstruction method, based on the principle of in-plane displacement measurement of the deformed transmission-virtual grating image, produced by 3-D shape of liquid surface, has been proposed. This study provides an effective method for further research on interactions between floating body and liquid surface.

References

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