EXPERIMENTAL DETERMINATION OF TIME-DEPENDENT MECHANICAL PROPERTIES OF SOFT MATERIALS

Yun-Che Wang*a), Chih-Chin Ko*, Yu-Ti Wu* & G. Nakamura**
*Department of Civil Engineering, National Cheng Kung University, Tainan, Taiwan, 70101
**Department of Mathematics, Hokkaido University, Sapporo, Japan, 060-0810

Summary Soft materials are widely used for biological or damping applications. In this work, experimental and data analysis methods are developed to measure the linear viscoelastic properties, in the frequency domain, of soft hydrogels with a pendulum-type viscoelastic spectroscopy apparatus. The apparatus is capable of measuring, in a relatively wide frequency range, dynamic bending modulus under pure bending excitation, and dynamic shear modulus under torsional excitation, as well as viscoelastic damping of the corresponding deformations. The Lorentzian curve-fit method was adopted to extract damping from the resonance data. Under wet conditions, the measured loss tangent was 0.12 for bending and 0.08 for torsion. For dry hydrogel, the loss tangent was 0.05 for both bending and torsion. The measured properties are compared with a commercial rheometer device to validate our data and to show the limitation of the commercial device.

INTRODUCTION

Mechanical properties of soft materials with low stiffness, such as polymer and hydrogel, are of importance for their roles in biomedical applications. A recent development on creating strong hydrogels with double networks has shown promising combined mechanical properties for applications to articular cartilage [1]. However, the mechanical properties of such materials are not fully studied, particularly for their linear viscoelastic properties. In this work, a pendulum-type viscoelastic spectroscopy was developed to measure complex modulus and loss tangent up to tens of kilo Hertz. Most viscoelastic instruments, such as DMA (dynamic mechanical analyzer), are not suitable for mounting soft samples. Also, their frequency ranges are usually not large enough to capture mechanical behavior in the high frequency range. In the present work, we focus on the capability of the experimental method to obtain viscoelastic properties at high frequency by the resonance method. The measured data may be compared with data from magnetic resonance elastography [2].

THEORETICAL

The dynamic behavior of a viscoelastic rod under torsion can be solved by using the elastic-viscoelastic corresponding principle. Starting with the governing equation for the elastodynamic torsion problem with the shear modulus μ and density ρ,

$$\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2} = \frac{\rho}{\mu}\frac{\partial^2 u_\theta}{\partial t^2}, \tag{1}$$

one obtains the normalized dynamic compliance with its maximum value being normalized to unity, called the Lorentzian curve-fit function, as below.

$$\Gamma = \sqrt{\frac{\tan^2\delta}{\left[1-(\omega/\omega_0)^2\right]^2 + \tan^2\delta}} \tag{2}$$

Here the viscoelastic damping is quantified as $\tan\delta$, driving angular frequency ω and resonant frequency ω_0.

EXPERIMENTAL

The pendulum-type viscoelastic spectroscopy apparatus is capable of applying torque for torsion tests and bending moment for pure bending test [3]. Applied force couple $\mathbf{T}$ is generated from the interaction between a permanent magnet $\boldsymbol{\mu}$ and a set of Helmholtz coils $\mathbf{B}$, i.e. $\mathbf{T} = \boldsymbol{\mu} \times \mathbf{B}$. Two sets of Helmholtz coils are manufactured; one for torsion and the other for bending tests. The permanent magnet, with a mass of 0.25 g, is glue-attached to the free-end of the specimen, and a mirror is put on the magnet to reflect the laser beam. The fix-end of the specimen is glued to a support rod. Mechanically, the specimen can be modeled as a cantilever beam with lumped mass at its free end. For quasi-static tests, DC voltage is used to generate unit step function with care on signal overshoots. To do dynamic studies, sinusoidal AC signals are applied to the Helmholtz coils. Deformation of the specimen is measured with a laser-based displacement

a) Corresponding author. E-mail:yunche@mail.ncku.edu.tw

measurement system consisting of a 5 mW diode laser and two-dimensional, silicon-based position sensor. The torsion twisting angle and bending angle at the free-end of the cantilever beam can be obtained from the measured displacement. In the sub-resonant regime, the phase lag between excitation and response can be measured directly. At the resonant peaks, mechanical properties can be deduced from the resonant frequency and the shape of the resonant peak by applying the Lorentzian curve fit.

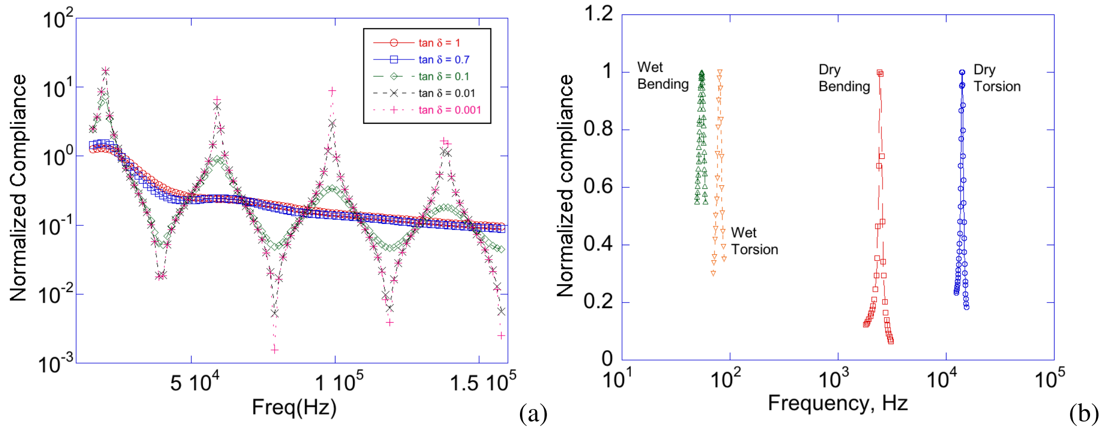

Figure 1. (a) Theoretical calculation of the resonant spectrum of the cantilever beam under torsion with various loss tangent. (b) Experiemental measured resonant peaks of the soft material under wet and dry conditions.

RESULTS AND DISCUSSION

Figure 1 (a) shows the calculated results of dynamic compliance with a set of chosen parameters (length L=40mm, diameter d = 3 mm, shear modulus μ= 27 GPa, density ρ=2700 kg/m^3) to demonstrate the effects of large damping. As can be seen, the oscillatory behavior disappears when loss tangent is greater than 0.7. Figure 1 (b) shows our experimental data near resonant peaks of the hydrogel in the wet and dry conditions. The geometry for the dry sample is 12.97 mm long, 4.81 mm wide and 3.32 mm thick, and that for the wet sample, 9.19 mm long, 9.5 mm wide and 8.24 mm thick. The density of the dry sample is about 960 kg/cm^3, and that of the wet same about 1000 kg/m^3. In the wet condition, the sample is fully moisturized, but no water flow. From the Lorentzian curve-fit on the torsion resonant peaks, the measured loss tangents are 0.08 (around 80 Hz) and 0.05 (around 14 kHz) for the wet and dry condition, respectively. From the bending resonant peaks, the measured loss tangent is 0.12 for the wet around 50 Hz. For the dry case, its bending peak reveals the same tangent delta as that of the wet case. The bending loss tangent larger than the torsion one for the wet sample may be due to thermoelastic damping arising in the bending mode near the driving frequency. Thermoelastic damping may be frequency and sample-geometry dependent. We have compared our measured data with a commercial TA rheometer. and reasonable agreement was obtained in the low frequency regime.

CONCLUSIONS

In summary, we have experimentally measured the linear viscoelastic properties of hydrogel from the resonant peaks up to hundred to thousand Hertz. In the low frequency regime, below 10 Hz, linear viscoelastic damping obtained from our apparatus is consistent with that from a commercial TA rheometer. Furthermore, in the tested high frequency range, the damping of wet hydrogel is roughly the same as the that in the low freuqency. In other words, for the wet hydrogel, its damping is on the order of high 10^{-2} in about 1 to 100 Hz.

References

[1] Gong J.P., Katsuyama, Y., Kurokawa, T., Osada, Y.: Double-Network Hydrogels with Extremely High Mechanical Strength. *Advanced Materials* **15**:1155–1158, 2003.

[2] Muthupillai, R., Lomas, D.J., Rossman, P.J., Greenleaf, J.F., Manduca, A., Ehman, R.L.: Magnetic Resonance Elastography by Direct Visualization of Propagating Acoustic Strain Waves. *Science* **269**:1854–1857, 1995.

[3] Lakes R.: Viscoelastic Materials, Cambridge University Press, NY, 2009.