APPROXIMATIONS OF INCOMPRESSIBLE LARGE DEFORMATION ELASTIC PROBLEMS: SOME UNRESOLVED ISSUES!

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Abstract

Many finite element formulations fail to properly reproduce highly incompressible finite strain solutions, showing unphysical instability hourglass modes. This has been a puzzling point for many researchers who have been devoted many efforts in trying to produce stable and robust numerical schemes also in a finite strain regime. However, up today there is no effective and exhaustive proof that any of the proposed approach is really working, due also to the fact that there is not yet a clear theoretical understanding on the numerical pathologies. We try to present some general considerations, highlighting that in some situations, not only enhanced schemes fail in reproducing the solution stability range, but that similar failures can be also observed when using mixed finite elements generally considered reliable.

INTRODUCTION

It is well known in the literature that many finite element formulations fail to properly reproduce highly incompressible finite strain solutions, showing unphysical instability hourglass modes [1]. This has been a puzzling point for many researchers who have been devoted many efforts in trying to produce stable and robust numerical schemes also in a finite strain regime. However, up today there is no effective and exhaustive proof that any of the proposed approach is really working, due also to the fact that there is not yet a clear theoretical understanding on the numerical pathologies. This observation is also evident from the numerical point of view, since there is no agreement on which could be effective numerical round-robin tests and on possible benchmark tests to really test and prove the robustness of approximation schemes.

PROPOSED APPROACH

As a consequence, within the framework of incompressible finite elasticity, we focus on some simple two-dimensional examples, reported in Figures **??**.

In particular, for the first problem we are able to study the stability of the continuum problem, while the second one seems to be a simple numerical problems; we may hope that in the future both problems may become benchmark tests.

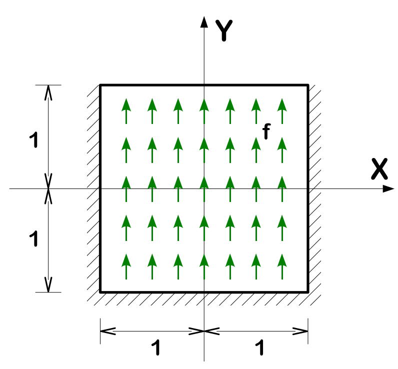

Figure 1. Problem 1.

Depending on the specific problem, we theoretically and numerically study the stability of the solutions to the discretized problem, obtained by means of displacement, mixed, mixed-enhanced finite elements, which are well-performing in linear elasticity (some results can be found in [2, 3]).

In particular, we explore the dependency of the numerical results on the specific form of the Θ function enforcing the incompressibily constraint. From a theoretical point the solution of the continuous problem is clearly independent from such a function, while we observe that several reliable approximation schemes show a dependency of the numerical solution on the adopted Θ function.

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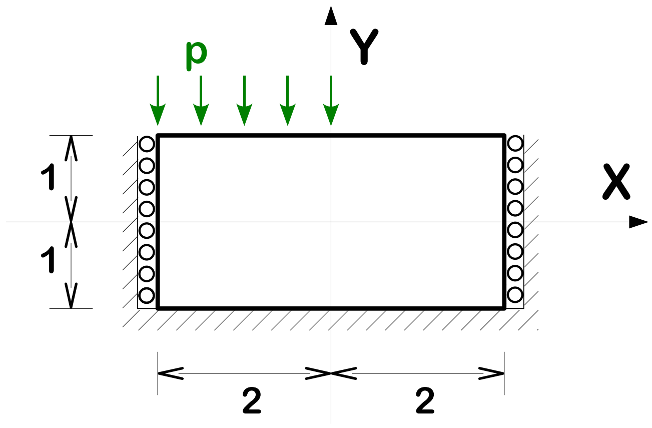

Figure 2. Problem 2.

Approx	Θ	Mesh	Load	v
Q2-P1	$(J-1)$	8 x 4	60.3347	0.116456
		16 x 8	60.5748	0.105804
		32 x 16	57.3658	0.098591
		64 x 32	55.2208	0.093658
	$Log(J)$	8 x 4	77.4181	0.161595
		16 x 8	69.2701	0.135808
		32 x 16	64.3658	0.115045
		64 x 32	62.5729	0.109898
	$1-1/J$	8 x 4	88.4547	0.211841
		16 x 8	73.1676	0.157548
		32 x 16	66.7125	0.124232
		64 x 32	63.462	0.111932

Table 1. Numerical results for the Q2-P1 approximation for problem 2

As an example, in Table we report some numerical results for the case of a quadrilateral element based on quadratic continuous approximation for the displacement and linear discontinuous approximation for the pressure ($Q2-P1$ element). In particular, we report the maximum load multiplier (Load) and vertical displacement of a point before detecting numerical instability. The results are reported for different meshes and for different Θ functions.

From an inspection of the numerical results it is clear that the adopted Θ function plays a significant role and that different instability loads are detected.

CONCLUSIONS

We conclude drawing some general considerations. In particular, we find out that, in some situations, not only enhanced schemes fail in reproducing the solution stability range, but that similar failures can be also observed when using mixed finite elements generally considered reliable.

References

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