A NEW STRATEGY FOR MODEL REDUCTION IN NON-LINEAR BIOMECHANICS

Stefanie Reese[*a)], Annika Radermacher[*]
[*]Department of Civil Engineering, RWTH Aachen University, D-52074 Aachen, Germany

Summary Simulations for surgery training programs or online support during surgeries require simulation tools which are characterized by a minimal simulation time and a high degree of accuracy. To achieve this, model reduction is needed. This is the significant reduction of the number of degrees-of-freedom of a system while preserving a prescribed level of accuracy. In the present paper, various singular value decomposition-based model reduction methods are discussed in the context of biomechanics. It turns out, that the proper orthogonal decomposition (POD) method is the most promising among the investigated approaches. It yields the best approximation of the displacement as well as of constitutively dependent variables such as stress. This is achieved while simultaneously providing considerable reduction in the computational effort.

INTRODUCTION

The functional endoscopic sinus surgery (FESS) represents the state of the art regarding minimally invasive techniques for treatment of paranasal sinusitis. The surgery restores the normal drainage and ventilation and ensures a constant flow of the mucus. A biomechanical model of the nasal cavity can expedite the development of the FESS by supplying important data like stress and deformation. Furthermore a surgery training program or an online support during the surgery can be built up from such a model. Simulations to support such applications should be performed in minimal computational time (possibly in real time) and provide results with a high level of accuracy. The constantly rising requirements concerning biomechanical simulations together with the nonlinearity of those systems lead to numerical models with an increasing number of degrees-of-freedom (dof). Therefore model reduction is needed to allow for the required real time simulation.
There exist different approaches to deal with this problem in the literature. In the medical field, one often uses spring mass (e.g. [7]) or tensor mass systems (e.g. [4]) for training simulations. These methods simplify the model by using springs or polynomial functions to describe the nonlinear material behavior. In other research fields like turbulence modeling, fluid dynamics, image processing or signal analysis many different model reduction methods exist. Based on [1] two classes of model reduction methods are identified: the singular value decomposition (SVD)-based methods and the Krylov-based methods.
This paper will focus on the SVD-based model reduction methods. These methods project the equation system on a subspace of smaller dimension. There is a variety of choices for this important subspace. The modal basis reduction method (e.g. [9]) developed in the field of dynamical structural analysis chooses the modal eigenforms as subspace. In the load-dependent Ritz method (e.g. [11]) these eigenvectors are approximated by the so-called Ritz vectors. Originating from the turbulence research field the proper orthogonal decomposition (e.g. [8, 3]) method builds up the subspace by using snapshots from a precomputation. The second group, the Krylov-based methods, employs moment matching of the transfer functions (e.g. [1]) to reduce the system. All concepts mentioned above were developed and are widely used for solving linear problems. There exist only a few strategies to expand these methods to nonlinear systems ([12] [6]). Especially in biomechanical applications *Niroomandi et al.* [10] and *Dogan and Serdar* [5] use proper orthogonal decomposition based (POD) methods. In contrast, the Krylov-based methods are recognized as being not useful for nonlinear problems [14].
In the present paper we extend the modal basis reduction, the load dependent Ritz and the proper orthogonal decomposition method to nonlinear elasticity and inelasticity including large deformations. The methods are tested for different material laws, a hyperelastic Neohookean and a viscohyperelastic model. The performance of these methods in the nonlinear context is first studied at the example of a cube under compression. Afterwards we investigate a more complex and realistic geometry, an inferior turbinate.

PROPER ORTHOGONAL DECOMPOSITION FOR NON-LINEAR BIOMECHANICS

The POD method (e.g. [2, 8, 3]) uses a certain data basis called snapshots to build up the so-called subspace matrix $\mathbf{\Phi}$. To collect the snapshots the full system has to be computed beforehand in a precomputation. The snapshots are the solution vectors of $\mathbf{G}\left(\mathbf{U}\right) = \mathbf{R}\left(\mathbf{U}\right) - \mathbf{P} = \mathbf{0}$ (1) where $\mathbf{R}\left(\mathbf{U}\right)$ is the residual force vector, $\mathbf{P}$ the external load vector and $\mathbf{U}$ the global vector of nodal displacements. The latter equation holds for any static finite element system. Linearizing equation (1) and multiplying the linearized equation by the subspace matrix yields

$$\underbrace{\mathbf{\Phi}^T\ \mathbf{G}(\mathbf{\Phi}\ \mathbf{U}_{red,i})}_{\mathbf{G}_{red}} + \underbrace{\mathbf{\Phi}^T\ \mathbf{K}_T(\mathbf{\Phi}\ \mathbf{U}_{red,i})\ \mathbf{\Phi}}_{\mathbf{K}_{T,red}}\ \Delta\mathbf{U}^j_{red,i+1} = \mathbf{0} \qquad (2)$$

depending on the so-called reduced diplacement vector $\mathbf{U}_{red}$ given by $\mathbf{U} = \mathbf{\Phi}\,\mathbf{U}_{red}$. The index j in (2) refers to the time step t_j. Note that the matrix $\mathbf{\Phi}$ with the dimension $m \times n$ is held constant during one time step. m is the dimension of the reduced system, n the dimension of the original system. We certainly strive for a situation characterized by $m \ll n$.

[a)] Corresponding author. E-mail: stefanie.reese@rwth-aachen.de

Cube Under Compression

To compare the three model reduction methods investigated here (modal reduction, load-dependent Ritz method and the POD), a cube under compression with large deformation is investigated. The model reduction methods are implemented into the finite element solver FEAP developed by Taylor [13].

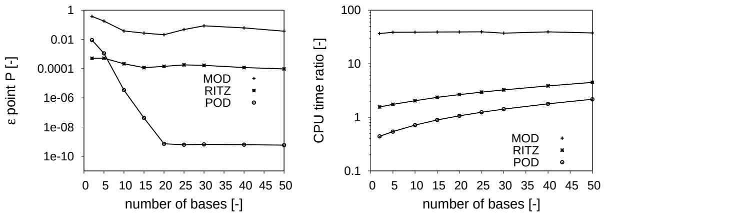

Figure 1. Error (a) and CPU time (b) for the methods MOD, RITZ and POD

Note that increasing the number of bases means to increase m. It is obvious from Fig. 1a that the POD method leads to the smallest error in the displacement at a special point P. Additionally, also comparison of the CPU time ratio (Fig. 1b) shows the most advantageous result for the POD.

Inferior Turbinate

In this section the POD method is used to reduce a more realistic biomechanical model. The geometry of an inferior turbinate pressed down during the FESS, is segmented from CT data. The segmented geometry is shown in Fig. 2a. The geometry is simplified by extruding an average cross section. The investigated model (Fig. 2b) has 6048 DOFs and is discretized by the same FEAP elements as the cube. The turbinate is modeled by means of the Neohookean material model from FEAP using the same material parameters as the cube. The applied load is a pressure load, simulating the endoscope impact. The turbinate is constrained on the side to the maxillary sinus.

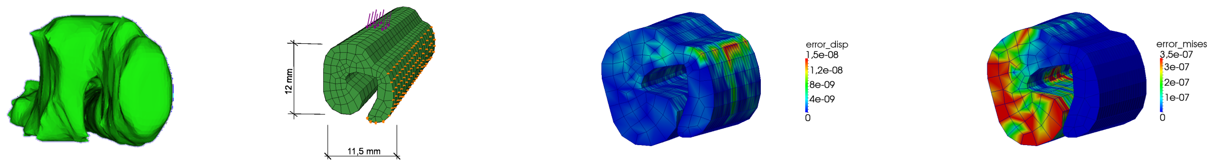

Figure 2. a) Segmentation; b) geometry and boundary condition; c) displacement error; d) stress error

To investigate the model reduction in context of the turbinate a relative error of the displacement and von Mises stress at a fixed time step is used. A contour plot of both errors for using five bases is shown in Fig. 2c. The largest relative errors are located where the absolute value of the corresponding variable has its smallest value. For a surgical application where the absolute value of the error is important these areas are negligible. The largest stress values in the turbinate are located near the constraints, where the stress error of the approximation is nearly zero (Fig. 2d). On the other side of the turbinate the displacement values reach their maximum and the displacement error their minimum. Therefore the POD method is able to approximate the behavior of this biomechanical system very well and simultaneously allows an enormous reduction in the computational effort.

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