THE FLOW PAST A LATTICE OF AIRFOILS AND THE DETERMINATION OF SEPARATION POINTS IN A VISCOUS FLUID

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Summary A numerical method for computing the potential flow past the lattice of airfoils is described. The problem is reduced to a linear integro-differential equation on the lattice contour, which is approximated with the help of specially derived quadrature formulas. The quadrature formulas exhibit exponential convergence in the number of points on an airfoil. The method can be used to optimize the airfoils as based on any given integral characteristics.

INTRODUCTION

Numerical methods of computing flows past lattices with separation points are well described in articles which have been referred in monograph [1] and article [2]. As opposed to previous works, in current work a more convenient integral equation is used. Furthermore, a new simple and precise approximation of integral equation by set of linear equations is applied. In order to determine the separation points the equations of the boundary layer are solved. The separation points are determined from the condition of tangential tensions' equality. Additionally, the condition of pressures' equality in the separation points is considered.

QUADRATURE FORMULAS FOR PERIODIC FUNCTIONS

For infinitely differentiable functions of period l, the quadrature formulas have an especially simple form [1]:

$$\int_0^l y(x)dx = \frac{l}{N}\sum_{j=1}^{N} y_j, \quad (1)$$

$$\int_0^l y(x)\ln|\sin\frac{\pi}{l}(x-x_i)|\,dx = \frac{l}{N}\sum_{j=1}^{N}\alpha(|i-j|)y_j,$$

$$\alpha(m) = -\left(\ln 2 + \frac{(-1)^m}{N} + \sum_{k=1}^{M-1}\frac{1}{k}\cos\frac{2\pi km}{N}\right),$$

$$M = \frac{N}{2}, \quad m = 0,1,\dots,N$$

It can be shown that these formulas are exact for trigonometric polynomials of the form $y(x) = \frac{a_0}{2} + \sum_{k=1}^{M} a_k\cos\frac{2\pi kx}{l} + b_k\sin\frac{2\pi kx}{l}$.

FLOW PAST A GRID OF BLADES

A periodic grid of blades is an infinite sequence of blades Lj for $j = \dots -n, \dots -1, 0, 1, 2, \dots, n, \dots$. The blade $Lj + 1$ is obtained by displacing Lj in the x direction through a distance of h (grid spacing) (see figure 1). Problems concerning flows past grids arise in turbine dynamics.

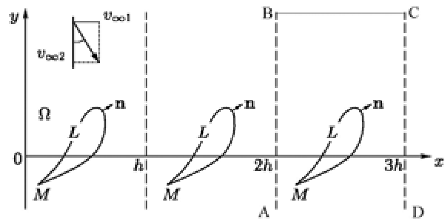

Figure 1

In order to calculate the velocity distribution on the profile boundary a system of equations is obtained.

$$A\,V(s) = 2\pi(C + xU_2 + yU_1) \qquad (2)$$

$$\int_0^l V(s)ds = -\Gamma, \quad U_1 - \frac{\Gamma}{2h} = v_{\infty 1}, U_2 = v_{\infty 2}$$

System consisting of four equations (2) with set circulations Γ and set velocities $v_{\infty 1}$ and $v_{\infty 2}$ on the inlet is enough to calculate velocity distribution $V(s)$ on the profile, the medial velocities U_1, U_2 and the constant C.
Here A is a linear integral operator

$$A\,V(s) = -\int_0^l G(s,s')V(s')ds'$$

where s and s' are coordinates of points M and M' on the profile, which are the lengths of arcs M_0M and M_0M' between the fixed point of blade M_0 and points M and M' accordingly.

System of linear equations

By means of quadrature formulas (1) system of equations (2) is approximated by the following linear system of equations

$$\sum_{j=1}^{N} A_{ij} V_j = 2\pi\left(C + x_i U_2 + y_i U_1\right), \quad \frac{l}{N}\sigma_j V_j = -\Gamma,$$

$$U_1 - \frac{\Gamma}{2h} = v_{\infty 1}, \quad U_2 = v_{\infty 2}$$

It is the system of N+3 linear equations for finding N values of the velocity on the profile $V_1, V_2, \ldots, V_N$,two medial velocities U_1, U_2 and constant C.

Having calculated $V_1, V_2, \ldots, V_N$, by means of the basic identity it is also possible to obtain flow function in any point beyond the profile

$$\Psi(x_i, y_j) = C + U_2 x_i + U_1 y_i + \frac{1}{2\pi N}\sum_{j=1}^{N} \sigma_j G_{ij} V_j$$

CALCULATION OF THE SEPARATION POINTS

If Reynolds number value $\mathrm{Re} = Lv_\infty / \nu$, is big enough, where *L* - is a characteristic size of a profile, v_∞ - is the fluid's velocity on an inlet, *ν* - a kinematic viscosity coefficient, Navier-Stokes equations can be approximated by the 'boundary layer' equations near the profile boundary.

These equations are essentially easier for numerical solution, than Navier-Stokes equations. Besides, there are one-parametric methods of its approximated solution build-up.

Kochina-Lojtsjansky and Lojtsjansky methods are the most effective methods in the case of a laminar boundary layer [2,3]. According to Lojtsjansky method a shearing stress *τ(x)* on the boundary can be analytically expressed through velocity of potential flow *V(x)*, where x is the length of the arc fixing the point on the blade, and shape parameter function.

It is useful to express the expression of the shearing stress through the dimensionless velocity $v(x) = V(x) / v_\infty$. Then the shearing stress will be

$$\frac{\tau}{\tau_0} = \alpha a^{\gamma-1/2}\left[v(x)\right]^{1+b/2}\left(f_1(x)\right)^{\gamma}\sqrt{L \Big/ \int_0^x v(\xi)^{b-1} d\xi},$$

$$\tau_0 = \frac{\rho v_\infty^2}{\sqrt{\mathrm{Re}}}, \quad a = 0.44, \quad b = 5.75, \quad \gamma = 0.74$$

In the case of laminar flow $\alpha a^{\gamma-1/2} = 1.07$.

Comparison of the exact and numerical solutions of the boundary layer equations for various profiles shows, that the offered formula's error is about 1%. Considering the flow past the plate we gain $\tau = 0.327\sqrt{\mu\rho v_\infty^{\ 2} / x}$. This result is in good correspondence with Blasius's exact formula $\tau = 0.332\sqrt{\mu\rho v_\infty^{\ 2} / x}$

This computational technique, combined with the boundary layer equations, has been used at the NPO Energomash to optimize water turbine blades. A detailed description can be found in [4,5]. On figure 2 numerical computations by our scheme (dashed line) are compared with direct numerical integration of Navier-Stokes equations (solid line) at the Reynolds number 20000.

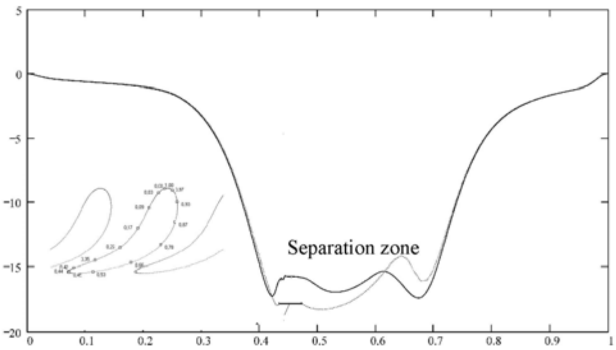

Figure 2

CONCLUSIONS

A highly precise scheme for flow past a lattice of airfoils is adduced. The comparison with another existing methods show our scheme to be rather accurate. The scheme is very common by not requiring the lattice construction. Due to high calculations speed one can optimize airfoils by any hydrodynamic properties with the help of our technique.

References

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