QUASI-CONFORMING ISOGEOMETRIC ANALYSIS FOR GEOMETRICALLY COMPLICATED DOMAINS

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Summary A quasi-conforming analysis for geometrically complex domains is proposed in this paper with a focus on nonmatching or trimmed CAD models. Geometrically exact interfaces and a degree of freedom reduction scheme can be applied without Langrage multipliers. The present method shares almost all advantages of isogeometric analysis and quasi-conforming finite element method (an assumed strain method) and is very suitable for the analysis of interfacial problems.

BACKGROUD

Recently, a trend demanding high-precision and high-performance computing and large scale applications is taking place in engineering analysis. Complex engineering structure often composed of multiple domains such as trimmed CAD models. Usually Finite element is only an approximation of the CAD geometry and this approximation can in many situations create errors in analytical results. Several typical examples related with interfacial problem of multiple domains may be mentioned: the contact problem, fluid-solid problem and boundary layer phenomena (aerodynamic and hydrodynamic problems) et al. In these fields, numerical results are sensitive to the exact geometry at the boundaries. Usually, for geometrically complex domains with common interface, Langrage multiplier method is employed to resolve those problems. The multiplier method such mortar method has the advantages of directly yielding a global solvable system while it must either satisfy the inf-sup conditions which necessitates special choices of multiplier spaces (such as mortar elements), or then stabilization techniques must be used. Another employed technique for handling interfacial constraint problems using direct solvers is to eliminate the Lagrange multipliers from the equation set to get a reduced set of degrees of freedom while this method is not always possible in a practical or efficient way.

Isogeometric analysis based on NURBS is introduced by Hughes et al. [1] and has been successfully applied in many fields. Isogeometric analysis selects a basis capable of exactly representing the known geometry and also as a basis for analysis so that the exact geometries can be employed to represent the common interfaces. For interfacial problem, exact geometry allows the boundaries are replaced using geometrically identical curves or surfaces, implying a degree of freedom reduction scheme can be applied using direct solver without Langrage multipliers. An assumed strain method named Quasi-conforming method was introduced by Tang et al [2]. It was presented as a general method that treats the conforming, non-conforming, and hybrid elements in a unified way. The key highlight is the inner integration on physical domains is replaced via boundary integration and the surface can be described using “string function”, by forming, a “net” within the surface or volume. By doing this, two or more subdomains can be joined together via common boundaries even if they share geometrically identical interfaces while their basis functions are nonconforming across the interface.

The purpose of this paper is to combine the existing quasi conforming technique and 3D solid NURBS-based finite element and attempt to apply it to geometrically complex domains including trimmed NURBS patches. The new method shares almost all the advantages of both of them:

- Exact geometry across the interface without smoothing issues.
- String function replaces the langrage multiplier without necessity to resort to iterative algorithm.
- Only shape functions themselves need to be computed, not their derivatives.

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Formula of quasi-conforming analysis using NURBS

In the quasi conforming technique, the surface is described using “string function”, by forming, a “net” within the surface or volume. By doing this, two or more patches can be joined together via boundary even if two CAD surfaces is not matching across their common boundaries. Firstly, we will discrete the strains (surface/displacement derivatives) in a NURBS element as shown in Fig.1. We assume the local strain should be

$$\varepsilon = \left[\frac{\partial \mathbf{u}}{\partial r}, \frac{\partial \mathbf{u}}{\partial s}, \frac{\partial \mathbf{u}}{\partial t}\right]^T \tag{1}$$

According to Taylor expansion, the items of strain can be assumed as follows:

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$$\frac{\partial \mathbf{u}}{\partial r} = \alpha_0 + \alpha_1 r + \alpha_2 s + \alpha_3 t + \alpha_4 r^2 + \cdots = \sum \alpha_i p_i = \mathbf{P}\boldsymbol{\alpha} \tag{2}$$

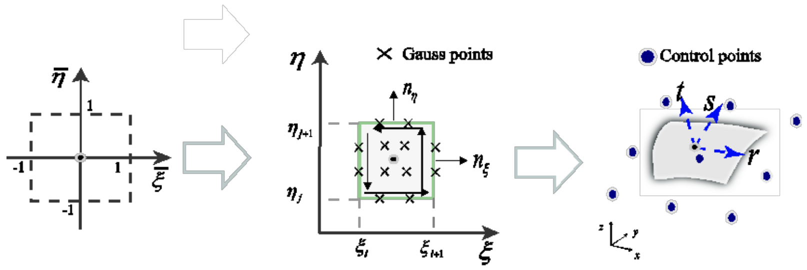

Fig.1: Integration and transformation of NURBS quasi-conforming elements.

Within a NURBS element, let the above approximation be

$$\int \mathbf{W}(\boldsymbol{\varepsilon}\text{-}\boldsymbol{\varepsilon}) = \mathbf{0} \tag{3}$$

where $\mathbf{W}$ is a weight function, defined as $\mathbf{P}^T$, usually, which guarantee the stiffness matrix symmetric. Rearranging.

$$\int_V \mathbf{P}^T \mathbf{P}\boldsymbol{\alpha} dV = \mathbf{A}\boldsymbol{\alpha} = \mathbf{Cu} = \int_V \mathbf{P}\frac{\partial \mathbf{u}}{\partial r} dV \tag{4}$$

where $\mathbf{Cu}$ can be evaluated using Green's theorem

$$\int_V \mathbf{P}^T \frac{\partial \mathbf{u}}{\partial r} dV = \oint_S \mathbf{u}\mathbf{P}^T n_r dS - \iiint_V \mathbf{u}\frac{\partial \mathbf{P}^T}{\partial r} dV \tag{5}$$

Then the final assumed strain is

$$\boldsymbol{\varepsilon} = \mathbf{J}\tilde{\boldsymbol{\varepsilon}} = \mathbf{JPA}^{-1}\mathbf{Cu}$$

Interfacial transform for geometrically complex domains.

In order to deal with multiple domains composed of the nonmatching or trimmed patches, we also design an interfacial transform algorithm to merge multiple surfaces together so that a direct assemble technique can be applied to quasi-conforming analysis.

CONCLUSIONS

A quasi-conforming analysis for geometrically complex domains is proposed in this paper with a focus on nonmatching or trimmed CAD models. Geometrically exact interfaces and a degree of freedom reduction scheme can be applied without Langrage multipliers. The present method shares almost all advantages of isogeometric analysis and quasi-conforming finite element method (an assumed strain method) and is very suitable for the analysis of interfacial problems.

Reference

[1] Hughes. T.J.R, Cottrell.J, Bazilevs,Y.: Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. Computer methods in applied mechanics and engineering. **194**:39-41, 2005.

[2] Tang L.M Chen W.J.: No. Quasi-conforming elements for finite element analysis. J Dalian Inst Technol. 10, 19–35.