INTERPRETATION OF A FRACTURE PHASE FIELD MODEL USING DISCRETE CONFIGURATIONAL FORCES

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Summary A phase field model formulation for brittle fracture is presented. The model can be understood as a regularized version of a sharp crack model with an additional crack field variable. The model is based on energetic considerations taking elastic and surface energy contributions into account. Furthermore the model incorporates dissipation by crack propagation via an evolution equation of non-local type. Based on these energy considerations the theory of configurational forces is applied to the model to reveal relations to a sharp crack, where energy arguments go back to the works of Griffith. For both, the phase field model and the configurational force balance, proper finite element implementations are presented.

FRACTURE PHASE FIELD MODEL

Model description

In their variational description of brittle fracture Francfort and Marigo [2] postulate a global minimization of the total energy with respect to any admissible crack set and displacement field as the only driving mechanism behind the crack evolution. The total energy consists of the elastic energy and the energy associated with the creation of new crack surfaces. Thus, a crack advances if this propagation is energetically favourable. This formulation is very strongly related to Griffith's energetic fracture criterion, but is more general, as it allows for crack path identification, crack branching and even crack initiation without additional criteria. However, a finite element discretization of the model is faced with serious technical problems, as it involves minimizations in a set of possibly discontinuous functions. To overcome this difficulty Bourdin [1] proposed a regularized version without discontinuities. As in damage models, the state of the material is characterized by an additional scalar field variable s in the regularized model. The value $s = 1$ is assigned to an undamaged material, while cracks are indicated by $s = 0$. The central part of the regularization is the formulation of a phase field energy density Ψ which incorporates the elastic energy and the surface energy of the cracks.

$$\Psi(\boldsymbol{\varepsilon}, s) = \left(s^2 + \eta\right) W(\boldsymbol{\varepsilon}) + \mathcal{G}_c \left(\frac{1}{4\epsilon}(1-s)^2 + \epsilon|\nabla s|^2\right) \quad \text{with} \qquad W(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\varepsilon} : (\mathbb{C}\boldsymbol{\varepsilon}) \tag{1}$$

In this work, the crack variable s is reinterpreted as an order parameter of a phase field model and cracking is addressed as a phase transition process. Therefore, the regularized model is supplemented by a Ginzburg–Landau type evolution equation for the crack field.

$$\dot{s} = -M\,\frac{\delta\Psi}{\delta s} = M\left[2\mathcal{G}_c\epsilon\,\Delta s - \left(2s\,W(\boldsymbol{\varepsilon}) + \frac{\mathcal{G}_c}{2\epsilon}(s-1)\right)\right] \tag{2}$$

where δ denotes the variational derivative. The Laplace-operator Δ incorporates a non-local character into the evolution equation. The Ginzburg–Landau equation can be regarded as a viscous approximation of Bourdin's formulation, see also [4].

Numerical implementation

The system of coupled field equations is formed by the physical force balance and the evolution equation. This system is solved using the finite element method. In addition to the nodal displacements, the crack field variable s is treated as an extra nodal degree of freedom. The implementation is done in 2D and 3D. For a robust implementation, the time integration of the evolution equation is done by means of an implicit Euler scheme. The non-linearity in the crack field potential requires a non-linear Newton iteration for the increments in each time step. In order to render useful results the mesh size has to be chosen carefully as the model has an intrinsic length scale. This point will also be addressed by some illustrative examples.

CONFIGURATIONAL FORCES

The solution of the coupled field equations of the phase field fracture model determines the entire fracture process. However, it is not obvious, that the evolution of cracks in the phase field model is consistent with the energetic considerations of the underlying variational framework. At this point, the concept of configurational forces can provide a better understanding and a graphic visualization of the energetic driving mechanisms of crack evolution in the phase field model, see Fig. 1.

In classical linear elastic fracture mechanics, configurational forces, also termed material forces, are well established and are often used to derive laws of crack propagation, see e.g. [3]. An adjustment of the configurational force concept to

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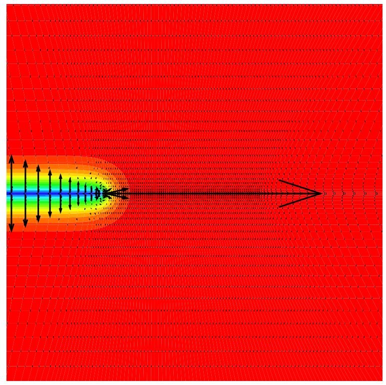

Figure 1. Contour plot of the crack field with configurational forces.

the phase field model yields similar kinetic relations for the evolution of diffuse cracks, see Fig. 2. In addition to the Eshelby stress tensor and the configurational volume forces of the classical configurational force balance, extra terms arise in the context of the phase field model due to the dependency of the phase field energy density Ψ on the crack field s. If interpreted appropriately, the different contributions of the configurational force balance in the phase field model can be related to classical fracture mechanical quantities, such as the energy release rate $\mathcal{G}$ and the threshold cracking resistance $\mathcal{G}_c$. In order to compute configurational forces, the extended configurational force balance is cast in a weak form by use of the same finite element discretization which is also used to solve the physical problem. For details on implementation issues the reader is referred to [5]. These relations as well as the kinetics of crack evolution in initiation and propagation scenarios are explored in numerical simulations, see Fig. 2 and 3.

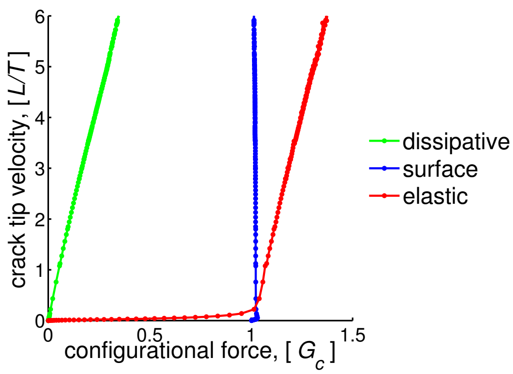

Figure 2. Kinetic relations.

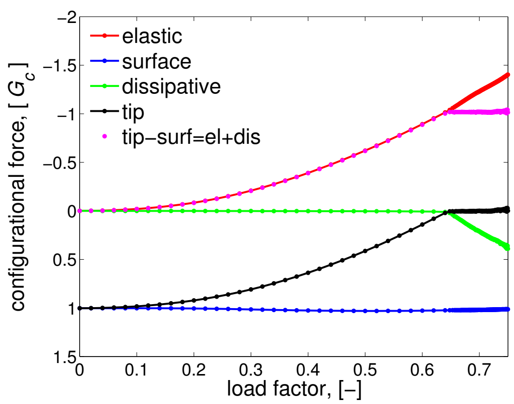

Figure 3. Configurational forces during mode I loading of an edge crack.

CONCLUSIONS

The combination of a fracture phase field model with the concept of configurational forces sheds more light on the relation of the fracture phase field model to the energetic considerations in fracture mechanics in the sense of Griffith. A consistent finite element implementation of the physical problem and the configurational force balance is readily obtained and allows the use of discrete configurational forces in the simulations. It must be noted that the configurational force balance in this setting is purely a post-processing technique. Nevertheless this post-processing is very attractive as it allows a more physical interpretation of the fracture phase field.

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