A PERIODIC FMM AND CALDERON'S PRECONDITIONING FOR ACOUSTIC-ELASTODYNAMIC COUPLED PROBLEMS

Hiroshi Isakari*, Naoshi Nishimura**a)
*Graduate School of Engineering, Kyoto University, Kyoto, 615-8540, Japan
**Graduate School of Informatics, Kyoto University, Kyoto, 606-8501, Japan

Summary A periodic fast multipole method (FMM) [1] for acoustic-elastodynamic coupled problems and its Calderon's preconditioning [2] are investigated. We consider a periodically perforated slab in water which is known to exhibit an extraordinary acoustic transmission [3]. The numerical results are verified with results from previous studies. It is also found that the Calderon preconditioning can accelerate the convergence of linear equation solvers more effectively than the conventional ones.

INTRODUCTION

Wave problems in periodic structures such as phononic crystals in acoustics have been brought to attention during the past few decades because of their remarkable features. Particularly interesting is a phenomenon called anomaly, in which a slight change of incident waves may cause drastic changes on the scattered fields. Near anomalies, the transmittance shows sudden peaks or dips as in the case of extraordinary acoustic transmissions. It is also known that the convergence of iterative solvers in FMM degrades near anomalies [1]. In this study, we shall construct an integral equation based solver for periodic wave problems related to phononic crystals which is fast even around anomalies. Namely, we propose a Calderon-preconditioned periodic FMM for acoustic-elastodynamic coupled problems and investigate its validity and efficiency through numerical experiments.

FORMULATION

Statement of the problem

Let D be a domain defined by $D = ((-\infty,\infty)\otimes[-\zeta_2/2,\zeta_2/2]\otimes[-\zeta_3/2,\zeta_3/2])$, which is further subdivided into two subdomains $D = D_1 \cup D_2$. We here assume that D_1 is an infinite domain composed of an inviscid fluid and D_2 is a finite domain filled with an elastic material, respectively. We consider doubly periodic problems in which the periodic lengths are ζ_2 along the x_2 axis and ζ_3 along the x_3 axis, respectively. The domain D is impinged upon by an incident sound pressure denoted by p_{I}. The sound pressure p is governed by the following Helmholtz' equation in D_1: $p_{,jj} + k^2 p = 0$, where $k = \omega\sqrt{\rho^{(1)}/\lambda^{(1)}}$ is the wavenumber of D_1, $\lambda^{(1)}$ is the bulk modulus and $\rho^{(1)}$ is the density. The displacement u_i is governed by the following Navier-Cauchy's equation in D_2: $\mu^{(2)}u_{i,jj} + (\lambda^{(2)}+\mu^{(2)})u_{j,ij} + \rho^{(2)}\omega^2 u_i = 0$, where $\rho^{(2)}$ is the density and $\lambda^{(2)}$, $\mu^{(2)}$ are the Lamé's constants. We assume the continuity of the traction and the normal component of the displacement rates as boundary conditions on the boundary ∂D between D_1 and D_2 and require the radiation condition to the scattered field in D_1. Furthermore, we require the periodic boundary conditions on the periodic boundary $S^{\mathrm{P}} = \{\boldsymbol{x}|\ |x_2| = \zeta_2/2 \text{ or } |x_3| = \zeta_3/2\}$.

Boundary integral equations and Calderon's preconditioning

The boundary integral equations which are equivalent to the above periodic boundary value problem are as follows:

$$\frac{1}{2}\left(p + \alpha\frac{\partial p}{\partial n_x}\right) = p_{\mathrm{I}} + \alpha\frac{\partial p_{\mathrm{I}}}{\partial n_x} + (\mathcal{S}+\alpha\mathcal{D}^*)\frac{\partial p}{\partial n_y} - (\mathcal{D}+\alpha\mathcal{N})p, \qquad \frac{1}{2}\boldsymbol{u} = \mathcal{T}\boldsymbol{u} - \mathcal{U}\boldsymbol{t}, \tag{1}$$

where α is the coefficient of the Burton-Miller method. Also, $\mathcal{S}$, $\mathcal{D}$, $\mathcal{D}^*$, $\mathcal{N}$, $\mathcal{U}$ and $\mathcal{T}$ are integral operators defined as follows:

$$\mathcal{S}v = \int_{\partial D} G^{\mathrm{P}}(\boldsymbol{x}-\boldsymbol{y})v(y)dS_y, \qquad \mathcal{D}v = \int_{\partial D}\frac{\partial G^{\mathrm{P}}(\boldsymbol{x}-\boldsymbol{y})}{\partial n_y}v(y)dS_y, \qquad (\mathcal{U}\boldsymbol{v})_j = \int_{\partial D}\Gamma_{jk}(\boldsymbol{x}-\boldsymbol{y})v_k(y)dS_y,$$

$$\mathcal{D}^*v = \int_{\partial D}\frac{\partial G^{\mathrm{P}}(\boldsymbol{x}-\boldsymbol{y})}{\partial n_x}v(y)dS_y, \quad \mathcal{N}v = \text{p.f.}\int_{\partial D}\frac{\partial^2 G^{\mathrm{P}}(\boldsymbol{x}-\boldsymbol{y})}{\partial n_x\partial n_y}v(y)dS_y, \quad (\mathcal{T}\boldsymbol{v})_j = \text{v.p.}\int_{\partial D}\Gamma_{Ijk}(\boldsymbol{x}-\boldsymbol{y})v_k(y)dS_y,$$

where "v.p." and "p.f." stand for Cauchy's principal value and the finite part of divergent integrals, respectively. G^{P} is Green's function which satisfies the periodic boundary conditions and the radiation condition. Also, Γ_{jk}, Γ_{Ijk} are the fundamental solution and the kernel of double layer of three dimensional elastodynamics, both of which satisfy the radiation condition.

We solve the following system of equations, composed of the boundary integral equations in Eq.(1) and the boundary

a) Corresponding author. E-mail: nchml@i.kyoto-u.ac.jp

conditions, numerically with iterative methods such as GMRES:

$$\underbrace{\begin{pmatrix} \dfrac{\mathcal{I}}{2} + \mathcal{D} + \alpha\mathcal{N} & -\rho^{(1)}\omega^2 \left(\mathcal{S} + \alpha \left(\mathcal{D}^* - \dfrac{\mathcal{I}}{2}\right)\right) \boldsymbol{n}_y^T \\ -\mathcal{U}\boldsymbol{n}_y & \dfrac{\mathcal{I}}{2} - \mathcal{T} \end{pmatrix}}_{\mathcal{A}} \begin{pmatrix} p(y) \\ \boldsymbol{u}(y) \end{pmatrix} = \begin{pmatrix} p_{\mathrm{I}}(x) + \alpha \dfrac{\partial p_{\mathrm{I}}(x)}{\partial n} \\ \boldsymbol{0} \end{pmatrix}, \tag{2}$$

where $\mathcal{I}$ is the identity operator. To improve the condition of the operator $\mathcal{A}$, we consider eliminating the hyper-singular integral $\mathcal{N}$ by virtue of the Calderon's formula [2] and a diagonal scaling. To this end, we consider the following operator:

$$\mathcal{M}^{-1} = \begin{pmatrix} -\dfrac{2\mathcal{S}}{\alpha} & \boldsymbol{0} \\ \boldsymbol{0} & \mathcal{I} \end{pmatrix}. \tag{3}$$

It can be proved that the eigenvalues of the operator $\mathcal{A}\mathcal{M}^{-1}$ accumulate around the three points given by $1/2, 1/2 \pm \mu^{(2)}/2(\lambda^{(2)} + 2\mu^{(2)})$ (> 0). We therefore expect that the use of the discretised $\mathcal{M}^{-1}$ as (the inverse of) preconditioner can accelerate the convergence of iterative methods. We note, however, that we have to replace $\mathcal{S}$ in Eq.(3) by its counterpart with a small frequency in order to avoid irregular frequencies.

NUMERICAL EXAMPLE

In order to verify the efficiency of the proposed approach, we consider the scattering by a periodically perforated tungsten slab immersed in water. We consider the normal incidence. The material and shape parameters for the slab are as illustrated in Fig. 1 (left). We divided the surface of the slab with 28054 triangular elements. Fig. 1 (centre) shows the transmittance curve versus normalised wavelength (wavelength in water divided by the period $\zeta_2 = \zeta_3 (= 1)$) Λ obtained by periodic FMM and the approximate analytical result by Estrada et al.[3], in which the slab is modeled as rigid body. The results obtained with the proposed method agree with the reference results except in the low frequency range. This is to be expected since the effect of the deformation of the slab is considered to be larger in low frequency. This figure shows the occurrence of an extraordinary acoustic transmission for wavelength larger than the period (i.e., $\Lambda \approx 1.22$). Also, the sudden dips of the transmission seen at $\Lambda = 0.45, 0.5, 0.7$ and 1.0 are anomalies. Even around the anomaly, the proposed method is accurate. To confirm the efficiency of the proposed preconditioning, we then compare the computational time of the proposed preconditioning approach with that of the conventional preconditioning method, in which we use the part of the matrix computed directly in the FMM algorithm as the right preconditioner. Fig. 1 (right) shows that the proposed Calderon preconditioning can accelerate the convergence more effectively than the conventional one. We also note that the proposed preconditioning is less affected by the anomaly than the conventional one.

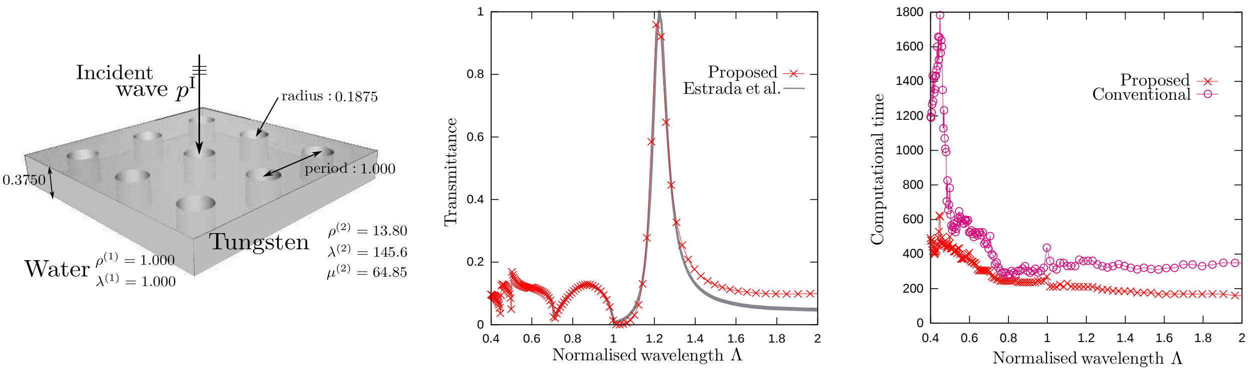

Figure 1. (left): Perforated slab immersed in water. (centre): Transmittance vs normalised wavelength. (right): Computational time vs normalised wavelength

CONCLUSION

We have proposed a periodic FMM and a Calderon preconditioning for acoustic-elastodynamic coupled problems. We have confirmed the validity and the efficiency of the proposed method through a numerical test.

References

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