A CARVING TECHNIQUE WITH THE AID OF POLYHEDRAL ELEMENTS AND SMOOTHED INTEGRATION FOR COMPLEX STRUCTURES

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Summary A novel mesh generation scheme is proposed with the aid of polyhedral elements. Soaking three-dimensional surface information into a structured mesh with hexahedral elements, the arbitrary polyhedral elements are obtained on the surface by trimming the hexahedral elements according to the surface information, while the interior of the domain is occupied by the structured hexahedral elements. Then, in conjunction with smoothed integration, the mesh consisting of the polyhedral elements and the structured hexahedral elements leads to highly efficient finite element analysis.

INTRODUCTION

Mesh generation of a three-dimensional structure with complex geometry still remains as a challenge since it is far from being straightforward to satisfy the nodal connectivity and the compatibility between the neighbouring elements for complex domains. Recently, Sohn et al. [1] proposed a three-dimensional mesh generation scheme with the aid of poly-pyramid elements, termed the so-called "carving technique". Their scheme consists of trimming and splitting processes. In the trimming process, surface information is soaked into a reference mesh structured with hexahedral elements, subsequently trimmed off are the elements intersecting with the surface. This results in the formation of the polyhedral elements on the surface. In the splitting process, the polyhedral elements are decomposed into several poly-pyramid elements of which shape functions are developed by moving least approximation [2–4]. The purpose of this work is to report on the carving technique with emphasis on its greatly improved version compared with that reported in the previous work [1]. Particularly, the splitting process has been eliminated, and the polyhedral elements are directly used with the aid of smoothed integration [4, 5], based on stabilized conforming nodal integration in meshfree methods [6]. The scheme in this work is extremely useful for treating domains with complex surface and interfaces such as a polycrystalline structure with multiple grains.

MESH GENERATION WITH POLYHEDRAL ELEMENTS

As shown in Fig. 1, the trimmed surfaces of the hexahedral elements are newly formed by connecting points of element edges intersecting with the model surface, and then triangulated because they may not be co-planar. As a result, we obtain the polyhedral elements with the trimmed surfaces, which are directly used in finite element analysis. For domains with several materials or with differently oriented crystallites in a polycrystalline material as shown in Fig. 2, the interfaces between the different materials or grains may act as some constraints on the mesh generation. Therefore, an additional treatment is introduced to overcome the constraints and to apply the aforementioned scheme for multiple materials or grains. Fig. 2(a) shows the polycrystalline structure with 8 grains Ω_i ($i = 1, 2, \ldots, 8$) and 20 grain boundaries $\Gamma_{ij} = \Gamma_{ji} = \Omega_i \cap \Omega_j$ between two neighboring grains. A line constraint formed in the region where three grains meet together, as indicated by $L_{ijk} = \Omega_i \cap \Omega_j \cap \Omega_k = \Gamma_{ij} \cap \Gamma_{jk}$. Similarly, a point constraint, $P_{ijkl} = \Omega_i \cap \Omega_j \cap \Omega_k \cap \Omega_l$, appears in the region where four different grains meet. To represent the grain boundaries appropriately without any gaps, the nodes of the hexahedral elements, closest to the line and the point constraints, are shifted to the location of the constraints, as shown in Fig. 2(b). By doing so, the aforementioned scheme is used as an efficient way to generate the mesh for the polycrystalline structure, as in the single material.

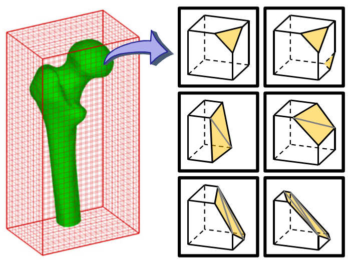

Fig. 1. Polyhedral elements throughout the trimming process.

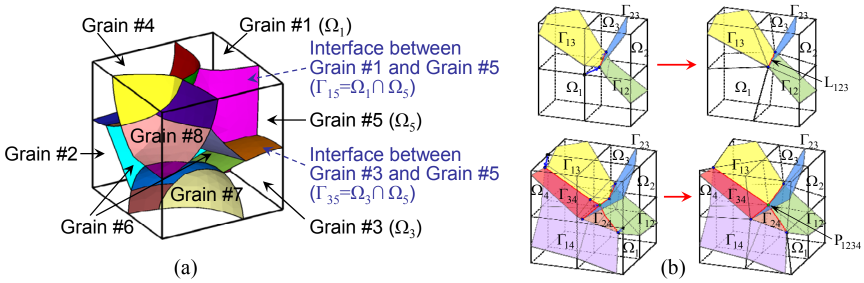

Fig. 2. A polycrystalline structures:
(a) grain boundaries between each two different grains; and
(b) adjustment of node positions for the domain with three and four grains.

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For complex domains with multiple materials or grains, the use of commercial tools requires an amount of substantial time in the procedure of mesh generation. The proposed scheme does not require the user-control time when constructing and identifying the closed surfaces for each grain to make them compatible with the adjacent grains. The mesh generation is automatically and efficiently conducted according to the given surface information of grain boundaries, satisfying the compatibility condition on the grain boundaries.

APPLICATION OF SMOOTHED INTEGRATION

Since it is very difficult not only to define the shape functions, but to implement numerical integration of the arbitrary polyhedral elements, the smoothed integration [4–6] is applied in this work. Using a smoothing operator and taking the divergence theorem, a volume integral in three-dimensional strain matrices is transformed into a surface integral. The smoothed integration leads to some important merits over Gauss integration in conventional FEM. It allows severe element distortion because physical-to-parental mapping is not required, and it is not necessary to explicitly define the shape functions because their derivatives are not used in the strain matrices with the smoothed integration [4, 5].

NUMERICAL EXAMPLES OF COMPLEX STRUCTURES

Consider a polycrystalline structure of $50 \times 50 \times 50\ \mu m^3$ cube of β-Ti, composed of total 8 grains. Fig. 3(a) shows the model, wherein 20 internal surfaces are formed by the grain boundaries. The displacement boundary conditions are imposed to represent uniform compression in the z-direction with the magnitude of 0.1 µm. The elastic moduli of β-Ti are given as three independent constants: $C_{11} = 97.7$ GPa, $C_{12} = 82.7$ GPa, and $C_{44} = 37.5$ GPa with reference to the crystal coordinate system, and each grain is randomly oriented.
The mesh configurations and the distributions of von Mises stress are shown in Fig. 3(b), which were obtained by the proposed scheme from 30×30×30 structured reference elements. To verify the accuracy and efficiency of the results with the polyhedral elements, the modeling and analysis of the same structure were also conducted with the poly-pyramid elements using the carving technique [1], with the hexahedral elements using voxel discretization, and with the tetrahedral elements using Abaqus. Fig. 3(c) shows the tendency of convergence investigated in terms of the total strain energy, as the number of nodes increases. The values from the proposed scheme are lower and closer to a reference solution with 10-node quadratic tetrahedral elements than those from the previous version of the carving technique, the voxel discretization, and Abaqus. The reference solution is chosen as a converged value, 13.769×10^{-9} J, calculated from Abaqus with 458,833 nodes and 330,699 elements. Fig. 3(c) shows that the proposed scheme provides a better solution in terms of the convergence and accuracy. More importantly, the present approach is amenable to fully automated generation of a mesh, enabling one to save a great deal of efforts in constructing meshes for complex geometry.

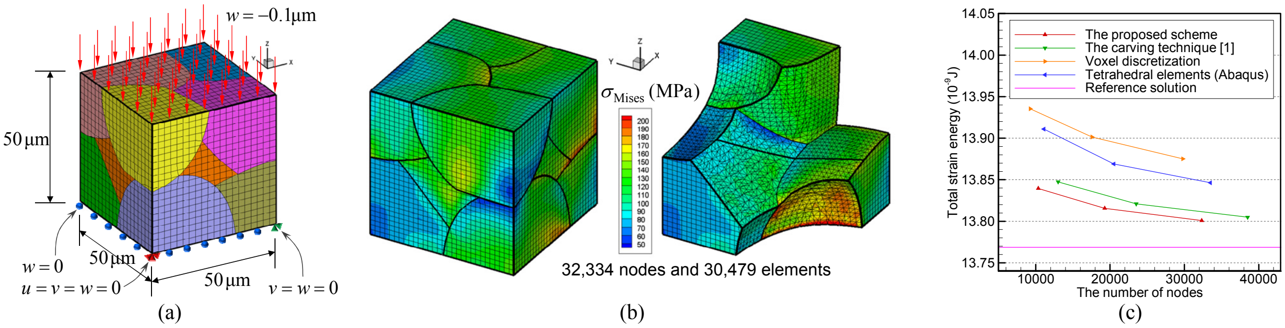

Fig. 3. The model description of the polycrystalline structure: (a) geometry and boundary conditions; (b) the distribution of von Mises stress in external and internal views; and (c) total strain energy versus the number of nodes.

CONCLUSIONS

An efficient scheme is presented to simplify procedures of the mesh generation and the analysis for complex domains. The proposed scheme makes it possible the automatic generation of meshes describing highly irregular surfaces or interfaces, and the application of the smoothed integration further improves the methodology in terms of the convergence and accuracy.

References

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