CONVOLUTION QUADRATURE BOUNDARY ELEMENT METHOD FOR 3D ELASTODYNAMIC ANALYSIS OF GENERAL ANISOTROPIC ELASTIC SOLIDS

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Summary It is known that a boundary element method (BEM) is an effective numerical approach for elastodynamic problems and some BEM applications have been published for transient problems of wave propagation and dynamic fracture of general anisotropic elastic solids. However, the conventional time-domain BEM has some disadvantages such that its formulation is complicated and the solutions sometimes become unstable. To overcome these disadvantages, the convolution quadrature boundary element method (CQ-BEM) for general anisotropic elastic solids is developed and some numerical examples are shown.

INTRODUCTION

Boundary element method (BEM) is well known as an effective numerical approach for wave propagation analysis. This method requires only boundary discretization, and can deal with an infinite or semi-infinite domain. In recent years, convolution quadrature boundary element method (CQ-BEM) has been developed by some researchers[1]. Though the CQ-BEM is a time-domain analysis, this method requires Laplace-domain fundamental solutions. Therefore CQ-BEM can produce stable solutions even for small time increments, which are not usually selected in the conventional time-domain BEM analysis.
In wave propagation analysis, anisotropic media has some features different from isotropic media. In an anisotropic solid, one polarized longitudinal wave and two different polarized transverse waves are generated. Therefore, 3D elastodynamic analyses of anisotropic solids are demanded to understand wave characteristics correctly. In this paper, the formulation of CQ-BEM for general anisotropic elastic solids is presented, and some numerical examples are shown. In general, a time-domain BEM for general anisotropic media requires much computational time, and it is said that this method is difficult to apply to a large scale problem. To overcome such a problem, a parallel computing method is implemented, and the validity of the present method and the contribution of parallelization are discussed.

CONVOLUTION QUADRATURE BOUNDARY ELEMENT METHOD

Convolution quadrature method

Convolution quadrature method (CQM) was proposed by Lubich[2]. CQM gives the numerical approximation for convolution integrals, which is written as follows:

$$f * g(n\Delta t) = \sum_{k=0}^{n} \omega_{n-k}(\Delta t) g(n\Delta t), \quad \omega_n(\Delta t) \simeq \frac{\mathcal{R}^{-n}}{L} \sum_{l=0}^{L-1} \hat{f}\left(\frac{\gamma(z_l)}{\Delta t}\right) \mathrm{e}^{-\mathrm{i}\frac{2\pi n l}{L}} \quad (n = 0, 1, 2, \cdots, N) \tag{1}$$

where Δt is the time increment and ω_n are the quadrature weights determined with the help of Laplace transform and a linear multistep method. In Eq.(1), $\hat{f}$ is the Laplace transform of f, and $\mathcal{R}$ is the CQM parameter. $\gamma(z_l)$ is the quotient of the generation polynomials, given as $\gamma(z) = \sum_{i=1}^{k}(1-z)^i/i$. This equation is derived by the backward differentiation formulas of order k, and we use this equation with $k = 2$. It is known that the CQM improves instability in a time stepping procedure.

Formulation of time-domain BEM using CQM

We consider a wave scattering problem in an infinite domain. Time-domain integral equations are derived by the elastodynamic reciprocal theorem, and written as follows:

$$u_i(\mathbf{x}, t) = u_i^{\mathrm{in}}(\mathbf{x}, t) + \int_S U_{ij}(\mathbf{x}, \mathbf{y}, t) * t_j(\mathbf{y}, t) dS(\mathbf{y}) - \int_S T_{ij}(\mathbf{x}, \mathbf{y}, t) * u_j(\mathbf{y}, t) dS(\mathbf{y}), \quad \mathbf{x} \in D \tag{2}$$

where **x** and **y** are the observation and source point. u_i and t_i are the displacement and traction, respectively, and u_i^{in} is the displacement of the incident wave. D and S are the infinite domain and its smooth boundary. U_{ij} and T_{ij} are the fundamental solution and its double layer kernel, respectively. In Eq.(2), taking limit process of $\mathbf{x} \in D \to \mathbf{x} \in S$, and discretizing with the CQM for time and with the collocation method for space, the discretized boundary integral equations can be obtained as follows:

$$\frac{1}{2} u_i^{\alpha}(n\Delta t) = u_i^{\mathrm{in}}(\mathbf{x}^{\alpha}, n\Delta t) + \sum_{\beta=1}^{M} \sum_{k=1}^{n} \left[A_{ij}^{n-k;\beta}(\mathbf{x}^{\alpha}) t_j^{\beta}(k\Delta t) - B_{ij}^{n-k;\beta}(\mathbf{x}^{\alpha}) u_j^{\beta}(k\Delta t) \right], \quad \mathbf{x}^{\alpha} \in S \tag{3}$$

where $\mathbf{x}^{\alpha}$ is the collocation point in α-th element, and $u_j^{\beta}(k\Delta t)$ and $t_j^{\beta}(k\Delta t)$ are the j-components of displacement and traction at the point $\mathbf{x}^{\beta}$ and the time $k\Delta t$. Considering the boundary conditions, we can obtain numerical solutions step by step in time.

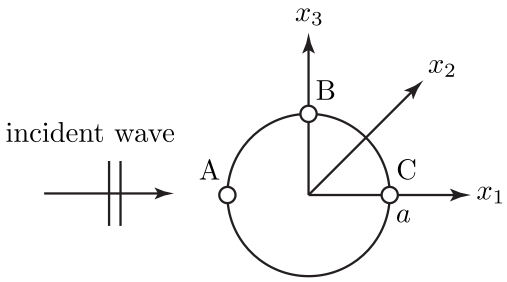

Figure 1. Model in 3D infinite domain

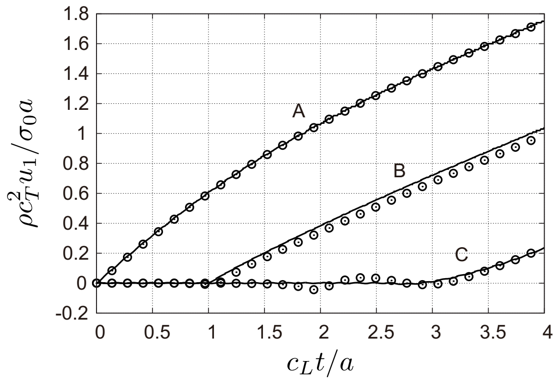

Figure 2. Time variations of displacements at point A, B and C in Fig.1

Influence functions $A_{ij}^{m;\beta}$ based on the CQM are written as follows:

$$A_{ij}^{m;\beta}(\mathbf{x}) = \frac{\mathcal{R}^{-m}}{L}\sum_{l=0}^{L-1}\left[\int_{S^\beta}\hat{U}_{ij}(\mathbf{x},\mathbf{y},s_l)\phi_t(\mathbf{y})dS(\mathbf{y})\right]\mathrm{e}^{-\mathrm{i}\frac{2\pi ml}{L}} \tag{4}$$

where $\hat{U}_{ij}$ is the fundamental solution in Laplace-domain, S^β is the boundary of the β-th element, and s_l is the discretized Laplace parameter. ϕ_t is the shape function corresponding to tractions. Replacing $\hat{U}_{ij}$ by $\hat{T}_{ij}$ and ϕ_t by ϕ_u, the influence functions $B_{ij}^{m;\beta}$ can be obtained in a similar manner. As seen in Eq.(4), the evaluation of boundary integrals can be done in the same way as that in a conventional Laplace-domain BEM. Additionally, the influence functions A_{ij}^{m} has the same form with discrete Fourier transform (DFT). If we select L as the same number of N, the fast Fourier transform (FFT) algorithm is available to speed up the computing.

NUMERICAL EXAMPLE

We consider a wave scattering problem in an infinite domain as a numerical example to check the validity of the proposed method by using isotropic parameters. The numerical model is a spherical cavity shown in Fig.1 and the sphere is divided into 512 elements. The time step $N(= L)$ is 32, and the time increment is given as $c_T\Delta t/a = 0.08$, where c_T is wave velocity of transverse wave. The incident wave is a plane longitudinal wave with a ramp function written as follows:

$$u_i^{\mathrm{in}}(\mathbf{x},t) = \delta_{i1}\frac{\sigma_0}{\rho c_L^2}(c_L t - x_1 - a)\cdot H(c_L t - x_1 - a) \tag{5}$$

where σ_0 is the stress amplitude, a is radius of the spherical cavity, c_L is wave velocity of longitudinal wave, and $H(\cdot)$ is Heaviside function. The time variations of the displacements at A, B and C in Fig.1 are shown in Fig.2. In this figure, the circles show the displacements computed by the present method, and the solid lines show the displacements obtained by Pao and Mow[3]. From these results, we can find that the circles almost agree with the solid lines. Therefore, it can be said that the validity of our method is confirmed. This computation is implemented via OpenMP as a parallelization scheme, which requires about 10 hours for computational time by using the OpenMP with 24 threads. In this computation, the supercomputer of Tokyo Institute of Technology, called TSUBAME2.0, is utilized.

CONCLUSIONS

We developed the convolution quadrature boundary element method (CQ-BEM) for a 3D elastic wave propagation problem of general anisotropic elastic solid, and a numerical example was shown. While the CQ-BEM is a time-domain analysis, the evaluation of boundary integrals is similar to that of Laplace-domain BEM. Therefore, the program coding is relatively easy compared with a conventional time-domain BEM. In addition, stable time-domain solutions can be obtained by using the CQM, even for small time increment which is not applicable in a conventional time-domain BEM. Though the OpenMP is implemented as a parallel computing method, much computational time and memory are still being required. In near future, computational time and memory will be reduced by using first multipole method (FMM) and MPI parallelization. This work was supported by KAKENHI (23760418).

References

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