SYMPLECTIC TIME-SUBDOMAIN ITERATIVE METHOD FOR NONLINEAR DYNAMIC EQUATION

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Summary Based on unconventional Hamilton variational principle in phase space for dynamics of multidegree-of-freedom system, a symplectic time-subdomain iterative method for nonlinear dynamic equations is proposed in this paper. Numerical example is presented to demonstrate the computational accuracy and efficiency of the method.

INTRODUCTION

There are two methods for nonlinear dynamic equation, analytic method and numerical method. Although analytic method is in the process of development, it could only be used to solve some special problems. Therefore, it is necessary to develop new numerical methods in order to meet the requirement of scientific research and the demands of engineering. Direct integration method (e.g., Newmark-β, Wilson-θ and Houbolt methods) has been widely applied in solving nonlinear dynamic equation, however, which has drawbacks such as energy dissipation and phase error. In addition, numerical method for initial value problems of ordinary differential equation (e.g., Runge-Kutta, Taylor, precise time integration [1] and Adams multi-step methods) can also be used to solve nonlinear dynamic equation. Nonlinear ordinary differential equation is transformed into non-homogeneous ordinary differential equation with constant coefficient bringing nonlinear term within non-homogeneous term when taking use of precise time integration method, which causes the complication of the solving process and the loss of accuracy. Symplectic time-subdomain algorithm based on unconventional Hamilton variational principle in phase space proposed by Luo [2], is unconditional stable method with simple concept, unified computational scheme and excellent computational performance.

In this paper, the nonlinear dynamic equation is separated into a linear equation and a nonlinear equation. Regarded the nonlinear term as load, a new method for nonlinear dynamic equation is proposed combined symplectic time-subdomain algorithm with iterative method. Numerical example is presented to indicate the validity and efficiency of the method.

SYMPLECTIC TIME-SUBDOMAIN ITERATIVE METHOD

Separation of nonlinear dynamic equation

The nonlinear dynamic equation for an N-degree of freedom (N-DOF) system is expressed as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}(t) + \mu\mathbf{f}(\mathbf{X}, \dot{\mathbf{X}}) \tag{1}$$

For weekly nonlinear system, the solution of Eq. (1) is considered as

$$\mathbf{X}(t) = \mathbf{X}_0(t) + \mathbf{X}_1(t) \tag{2}$$

Substituting Eq. (2) into Eq. (1), the original nonlinear dynamic equation is separated into a linear equation and a nonlinear equation after some rearrangement of terms as follows

$$\mathbf{M}\ddot{\mathbf{X}}_0 + \mathbf{C}\dot{\mathbf{X}}_0 + \mathbf{K}\mathbf{X}_0 = \mathbf{F}(t) \quad (\mathbf{X}_0(0) = a_0, \dot{\mathbf{X}}_0(0) = b_0) \tag{3}$$

$$\mathbf{M}\ddot{\mathbf{X}}_1 + \mathbf{C}\dot{\mathbf{X}}_1 + \mathbf{K}\mathbf{X}_1 = \mu f(\mathbf{X}_0 + \mathbf{X}_1, \dot{\mathbf{X}}_0 + \dot{\mathbf{X}}_1) \quad (\mathbf{X}_1(0) = 0, \dot{\mathbf{X}}_1(0) = 0) \tag{4}$$

Symplectic time-subdomain method for linear dynamic equation

When initial conditions are satisfied, the functional of unconventional Hamilton variational principle in phase space for linear dynamic equation (3) can be represented as

$$\prod(\mathbf{p},\mathbf{q}) = \int_0^{t_1} \left\{ \mathbf{p}^T\dot{\mathbf{q}} - \frac{1}{2}\mathbf{p}^T\mathbf{M}^{-1}\mathbf{p} - \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q} - \overset{\circ}{\dot{\mathbf{q}}}{}^T\,\mathbf{C}\mathbf{q} + \mathbf{F}^T\mathbf{q} \right\} dt + \tilde{\mathbf{p}}_0{}^T\tilde{\mathbf{q}}_0 - \overset{\circ}{\mathbf{p}}_1{}^T\,\mathbf{q}_1 \tag{5}$$

where $\mathbf{p}$ is momentum vector and $\mathbf{q}$ is displacement vector of linear dynamic system represented by Eq. (3), respectively.

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The variables with the superscript $\circ$ are the restricted variational variables and with the superscript ~ are initial values. The computational formulae[2] will be derived from $\delta\Pi = 0$, which can be used to solve $\mathbf{X}_0$ and $\dot{\mathbf{X}}_0$ of Eq. (3).

Iterative method for nonlinear dynamic equation

The nonlinear term $\mu\mathbf{f}(\mathbf{X}_0+\mathbf{X}_1,\dot{\mathbf{X}}_0+\dot{\mathbf{X}}_1)$ is described as

$$\mu\mathbf{f}(\mathbf{X}_0+\mathbf{X}_1,\dot{\mathbf{X}}_0+\dot{\mathbf{X}}_1)=\mu\mathbf{f}(\mathbf{X}_0,\dot{\mathbf{X}}_0)+\mu\mathbf{X}_1\cdot\frac{\partial\mathbf{f}}{\partial\mathbf{X}_0}+\mu\dot{x}_1\cdot\frac{\partial\mathbf{f}}{\partial\dot{\mathbf{X}}_0}+\mu O(\sqrt{\mathbf{X}_1^2+\dot{\mathbf{X}}_1^2})\approx\mu\mathbf{f}(\mathbf{X}_0,\dot{\mathbf{X}}_0) \tag{6}$$

Substituting $\mathbf{X}_0$ and $\dot{\mathbf{X}}_0$ solved from Eq. (3) into Eq. (6), Eq. (4) becomes

$$\mathbf{M}\ddot{\mathbf{X}}_1^*+\mathbf{C}\dot{\mathbf{X}}_1^*+\mathbf{K}\mathbf{X}_1^*=\mu\mathbf{f}(\mathbf{X}_0,\dot{\mathbf{X}}_0) \tag{7}$$

Eq.(7) is similar with Eq. (3) if regarded the right term as load, which means $\mathbf{X}_1^*$ and $\dot{\mathbf{X}}_1^*$ can be obtained by the symplectic time-subdomain. In the same way, a *n*th iterative scheme is given as

$$\mathbf{M}\ddot{\mathbf{X}}_{\mathrm{n}}^*+\mathbf{C}\dot{\mathbf{X}}_{\mathrm{n}}^*+\mathbf{K}\mathbf{X}_{\mathrm{n}}^*=\mu\mathbf{f}(\mathbf{X}_0+\ldots+\mathbf{X}_{\mathrm{n}}^*,\dot{\mathbf{X}}_0+\ldots+\dot{\mathbf{X}}_{\mathrm{n}}^*)-\mu\mathbf{f}(\mathbf{X}_0+\ldots+\mathbf{X}_{\mathrm{n-1}}^*,\dot{\mathbf{X}}_0+\ldots+\dot{\mathbf{X}}_{\mathrm{n-1}}^*) \tag{8}$$

$\mathbf{X}_0+\mathbf{X}_1^*+\mathbf{X}_2^*+\cdots+\mathbf{X}_n^*$ is the *n*th approximate solution of original nonlinear dynamic equation. It can be proved that the iterative process is convergent, and the proof is omitted here. Then, the obtained end values are also regarded as the initial values of the next time-subdomain and the iteration will be repeated until the values of the final time-subdomain are obtained.

NUMERICAL EXAMPLES

Calculating Van der Pol equation $\ddot{x}+x=0.2(1-x^2)\dot{x}$ with initial conditions $x_0=4$ and $\dot{x}_0=0$ by the proposed symplectic time-subdomain iterative method (STSI) and Runge-Kutta method (R-K).

The results are shown in Table 1, the data in the parentheses are the relative percentage errors. Table 1 shows that the results of STSI with time-subdomain Δt and $2\Delta t$ approach the results of R-K well and the relative percentage error is completed within 0.8%. While the length of time-subdomain increases to $5\Delta t$, the results of STSI still keep the higher accuracy and the CPU time can save 24% than R-K's. Additionally, Fig. 1 shows that the phase trajectory and partial enlargement of phase trajectory nearby point (2, 0). It can be seen that the energy dissipation of STSI is apparently smaller than R-K's, which means STSI has the superior ability of long-term tracking than R-K.

Table 1 Numerical results of variable *x* (Δt=0.1s)

Time (s)	R-K	STSI		
	Δt	Δt	2Δt	5Δt
2	1.2350	1.2374 (0.19)	1.2390 (0.31)	1.2384 (0.27)
4	-2.4429	-2.4467 (0.15)	-2.4494 (0.26)	-2.4569 (0.57)
6	0.7653	0.7612 (0.53)	0.7635 (0.24)	0.7737 (1.09)
8	1.5032	1.5123 (0.60)	1.5149 (0.77)	1.5194 (1.07)
10	-2.0131	-2.0100 (0.15)	-2.0105 (0.13)	-2.0122 (0.04)
20	1.9694	1.9586 (0.54)	1.9591 (0.51)	1.9604 (0.45)

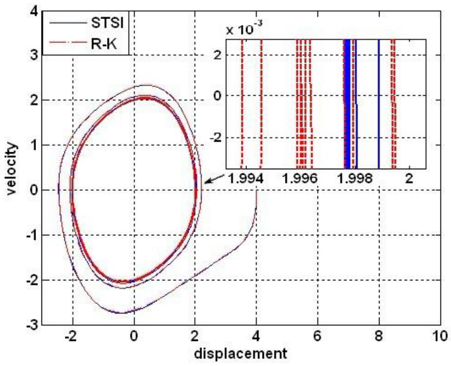

Fig. 1 Phase trajectory

CONCLUSIONS

Symplectic time-subdomain iterative method for nonlinear dynamic equation is proposed in this paper, which is the inheritance and development of symplectic time-subdomain. Numerical example was presented and compared to Runge-Kutta method to demonstrate the efficiency and accuracy of the proposed method. From the result of numerical example, it can be seen that the method is a highly efficient method with better computational performance and superior ability of long-term tracking, and it may have wide applications.

References

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[2] Luo En, HUANG Weijiang & ZHANG Hexin. Unconventional Hamilton-type variational principle in phase space and symplectic algorithm. Science in China (Series G), 2003, 46(3): 248-258