NONCONFORMING ISOGEOMETRIC ANALYSIS USING TC++ SPLINES

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Summary This paper addresses a long-standing challenge of applying isogeometric analysis to trimmed NURBS patches on the geometrically complex parametric domain in the field of isogeometric analysis. Emphasis is placed on the construction of a new analytical formula for the trimmed NURBS patch enriched via a set of nonconforming variables and TC++ spline basis functions, by which the homogeneous Dirichlet boundary conditions is satisfied and constructing analysis suitable geometry for geometrically complex domains is avoidable. The present method is hopeful to fill the gap of the CAD/CAM and FEA and give fresh impetus to the large scale industrial applications of isogeometric analysis.

BACKGROUD

Isogeometric analysis [1] is a new computational framework using regular NURBS patches not only for the geometric description but also for the field variables, which is hopeful to fill the gap existing between CAD/CAM and FEA. Non-uniform rational B-spline (NURBS) is the standard freedom surface representation in CAD/CAM. Their parametric presentation and the exact expression of conical sections is a big advantage. Despites its attractive geometric properties, the strict topology lead to many restrictions. In order to overcome the problem to create complex objects, trimming is an important operation in geometric modelling. Via Boolean operations, the superfluous regions of a surface are cut out and different parts with trimmed topologies can be tessellated into a complicated model. However, the stability is lost and the homogenous boundary condition cannot be satisfied if the regular domain is trimmed into a boundary domain when applying isogeometric analysis since the trimmed NURBS is not conforming to the boundary that, usually, represented using parametric trimming curves. This has become a major obstacle to using isogeometric analysis to the large scale industrial analysis.

Several solutions to single trimmed patches have been put forward while there is not much work, to the best of our knowledge, in the literature of isogeometric analysis on multiple trimmed NURBS patches. The best solution from a theoretical standpoint is to use the weight function, which is particularly suited for domains constructed from simple primitives with Boolean operations. Hollig presented the trimmed spline analysis using Web-Spline [2] based on R-function for a single trimmed patch. Sanches [3] et al. proposed the immersed b-spline finite element method using an implicit shape representation also based on weight function. However, it is difficult to construct weight functions for parametric domains though weight basis functions are stable and homogenous boundary condition can be matched exactly. Lagrange multiplier techniques can circumvent the direct discretization of boundary conditions. Hence, they do not need the meshes which conform to the boundary. Kim et al.[4] applied isogeometric analysis directly to a trimmed NURBS patch with multiple trimming curves and deal with boundary conditions using Lagrange multiplier method. The price paid is often Lagrange multiplier must either satisfy the inf-sup conditions which necessitates special choices of multiplier spaces, or then stabilization techniques must be used. The problems are unavoidable for two or more trimmed patches since multiple patches have no common interfaces to transfer the interface continuity.

TC++ SPLINE BASED NONCONFORMING ISOGEOMETRIC ANALYSIS

We present a new method, *nonconforming isogeometric analysis,* based on TC++ splines definite on irregular parametric domains. The present method still stays with the standard Galerkin framework, proposing a new type of finite element subspace. The highlights can be outlined as follows:

- Nonconforming control variables and nonconforming basis functions are introduced. Nonconforming basis functions vanished on the boundaries.
- Exact fulfilment of homogenous boundary conditions.
- Incorporation of trim curves/surfaces as the boundary presentation which is conforming to the inner domain. The control variables of trimmed curves/surfaces become a part of those of the trimmed patch.
- Compatibility with CAD/CAM applications.
- Two or more trimmed patches can be merged together easily.
- New analytical presentation for the trimmed patch.

TC++ Splines means Trimming Curve Plus Plus Splines, a new representation for trimmed patches via incorporating the control variables of trim curves/surfaces. As shown in Fig.1, a trimmed NURBS surface is composed of two components: a tensor product parametric surface and a set of properly oriented trimming curves. The basis functions of trimmed patches can be separated into three types: (a). Complete basis functions N_A. (b) Trimmed basis functions N_B. (c). Void basis functions N_V.

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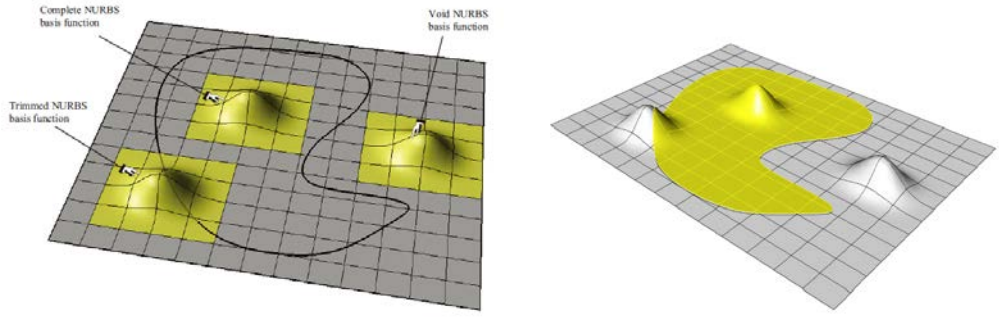

(a) **(b)**

Fig.1 (a) Complete NURBS basis functions, Trimmed NURBS basis functions and void NURBS basis functions. (b). Basis functions after trimming operation.

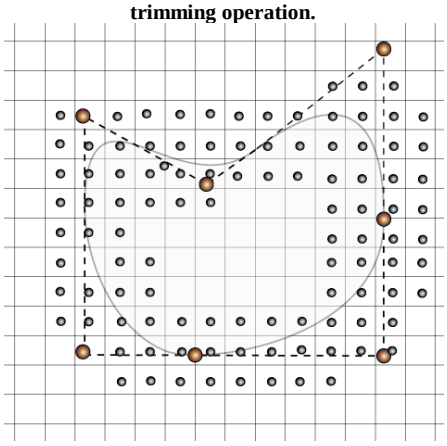

Fig.2 Control points of a NURBS patch related to the boundaries and control points of trimming curves.

The control variable related to those basis functions can also be divided into three collections: $\boldsymbol{d}_A$, $\boldsymbol{d}_B$ (black points, see Fig.2) and $\boldsymbol{d}_V$. As shown in Fig.2, the yellow points are the control points of the trimming curve, denote their control variables as $\boldsymbol{d}_C$. A special transformation matrix $\boldsymbol{T}$ of the control variables is introduced as follows:

$$\begin{pmatrix} \boldsymbol{d}_A \\ \boldsymbol{d}_B \end{pmatrix} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{T} \end{pmatrix} \begin{pmatrix} \boldsymbol{d}_A \\ \boldsymbol{d}_{B^*} \\ \boldsymbol{d}_C \end{pmatrix} \tag{1}$$

where $\boldsymbol{d}_{B^*}$ is the collection of the nonconforming control variables.

The displacement functions can be

$$\sum_A N_A d_A + \sum_B N_B d_B = \sum_A N_A d_A + \sum_{B^*} N^*_{B^*} d_{B^*} + \sum_{C^\Delta} N^\Delta_C d_C \tag{2}$$

where $N^*_{B^*}$ is the nonconforming basis function which is the linear combination of trimmed basis functions.

CONCLUSIONS

A new isogeometric method named *nonconforming isogeometric analysis* based on TC++ Splines is proposed, which exactly satisfies the homogenous boundary conditions. A set of nonconforming control variables are introduced to enrich the approximated space. The control variables of trim curves/surfaces are incorporated. Two or more trimmed NURBS patches can be merged together by TC++ spline, which make it possible to apply isogeometric analysis to large scale industrial applications.

Reference

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